Ratio Test

The **Ratio Test** is especially useful for series involving factorials, exponentials, or powers of n. For a series $\sum a_n$, compute

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|.$$

Then:

$L < 1$	the series converges absolutely,
$L > 1$ or $L = \infty$	the series diverges,
$L = 1$	the test is inconclusive.

In practice, the Ratio Test checks whether the terms are shrinking quickly enough from one term to the next for the series to converge.

Determine whether each series converges absolutely, converges conditionally, or diverges using the Ratio Test.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n^2}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^n}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{3^n}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n \cdot n!}{n^n}$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n! \cdot n^n}$
9. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n}{2^n}$
10. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n^2}{n!}$