Absolute Convergence

A series $\sum a_n$ is said to **converge absolutely** if the series of absolute values $\sum |a_n|$ converges. Absolute convergence is important because it guarantees that the original series $\sum a_n$ also converges. If $\sum a_n$ converges but $\sum |a_n|$ diverges, then $\sum a_n$ is called **conditionally convergent**.

For each of the following series, determine whether it converges absolutely, converges conditionally, or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n^2+1)}{n^3}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)\ln(n+1)}$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n)}{n^{3/2}}$
9. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$
10. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$