## Alternating Series Test

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

   Let $b_n = \frac{1}{n}$. Then:

   - $b_n > 0$, decreasing
   - $\lim_{n \to \infty} b_n = 0$

   The series converges by the Alternating Series Test.

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

   Let $b_n = \frac{1}{n^2}$. Then:

   - $b_n > 0$, decreasing
   - $\lim_{n \to \infty} b_n = 0$

   The series converges by the Alternating Series Test.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

   Let $b_n = \frac{1}{\sqrt{n}}$. Then:

   - $b_n > 0$, decreasing
   - $\lim_{n \to \infty} b_n = 0$

   The series converges by the Alternating Series Test.

4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

   Let $b_n = \frac{1}{\ln(n+1)}$. Then:

   - $b_n > 0$, decreasing
   - $\lim_{n \to \infty} b_n = 0$

   The series converges by the Alternating Series Test.

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$

   Let $b_n = \frac{n}{n+1}$. Then:

   - $\lim_{n \to \infty} b_n = 1 \neq 0$

   The Alternating Series Test does not apply. The series diverges by the Test for Divergence.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+(-1)^n}$

   Let $b_n = \dfrac{1}{n+(-1)^n}$. Then:

- $b_n$ is not decreasing, because the even and odd terms alternate:

$$b_{2k} = \frac{1}{2k+1}, \qquad b_{2k+1} = \frac{1}{2k}$$

  so, for example, $b_{2k} > b_{2k+1}$.

The Alternating Series Test does not apply.

7. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$

   Let $b_n = \frac{1}{n^{1/3}}$. Then:

   - $b_n > 0$, decreasing
   - $\lim_{n\to\infty} b_n = 0$

   The series converges by the Alternating Series Test.

8. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + \ln n}$

   Let $b_n = \frac{1}{n^2+\ln n}$. Then:

   - $b_n > 0$, decreasing for $n \geq 2$
   - $\lim_{n\to\infty} b_n = 0$

   The series converges by the Alternating Series Test.

9. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot \sin(1/n)}{n}$

   Let $b_n = \frac{\sin(1/n)}{n}$. Then:

   - $b_n > 0$
   - $\lim_{n\to\infty} b_n = 0$
   - To show $b_n$ is decreasing for $n \geq 1$, define $f(x) = \frac{\sin(1/x)}{x}$. Then

     $$f'(x) = -\frac{\cos(1/x)}{x^3} - \frac{\sin(1/x)}{x^2} < 0 \text{ for } x \geq 1,$$

     so $b_n = f(n)$ is decreasing.

   The series converges by the Alternating Series Test.

10. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n^{0.9}}$

    Let $b_n = \frac{1}{n^{0.9}}$. Then:

    - $b_n > 0$, decreasing
    - $\lim_{n\to\infty} b_n = 0$

    The series converges by the Alternating Series Test.