Remainder Estimates

Alternating Series Remainder

- **Applies to:** A convergent alternating series.
- **Remainder estimate:** The error when approximating the sum by the first n terms satisfies:

$$|R_n| = |S - S_n| \leq b_{n+1}$$

Integral Test Remainder

- **Applies to:** A convergent series $\sum a_n$, where $a_n = f(n)$, and $f(x)$ is a positive, continuous, decreasing function.
- **Remainder estimate:** If $S_n = \sum_{k=1}^{n} a_k$, then the remainder $R_n = S - S_n$ satisfies:

$$\int_{n+1}^{\infty} f(x)\,dx < R_n < \int_{n}^{\infty} f(x)\,dx$$

For Problems 1–5, use the Alternating Series Remainder Theorem. For Problems 6–10, use the Integral Test Remainder estimate. Show all reasoning.

Alternating Series Remainder

1. Approximate $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ using the first 4 terms. Estimate the error.
2. Approximate $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n}}{n^3+1}$ using the first 3 terms. How accurate is your approximation?
3. Find how many terms are needed to approximate $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ to within 0.0001.
4. Use the first 5 terms of $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n+1)}{n^2}$ to approximate the sum. Estimate the error.
5. Determine the minimum number of terms needed to estimate $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n}}{n \ln n}$ with error less than 0.01.

Integral Test Remainder

6. Approximate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ using the first 5 terms. Use the Integral Test to bound the error.
7. Use the Integral Test to estimate the remainder when approximating $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ by its partial sum through $n = 10$.
8. Estimate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ using the first 8 terms, and bound the error using the Integral Test.
9. Find the smallest n such that the partial sum $\displaystyle\sum_{k=2}^{n} \frac{1}{k(\ln k)^2}$ approximates the infinite series to within 0.05.
10. Approximate $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$ using the first 6 terms. Estimate how close your approximation is to the true value.