Integral Test

Integral Test. Suppose $a_n = f(n)$, where f is positive, continuous, and decreasing for all sufficiently large x. Then

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_1^{\infty} f(x)\,dx$$

either both converge or both diverge. This test is most useful when the terms of the series come from a function that can be integrated more easily than the series can be summed.

Use the Integral Test to determine whether each series converges or diverges.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$
2. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$
3. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)(\ln \ln n)}$
4. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ (general case)
6. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{\ln n}}{n}$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n^4+25}$
9. $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{n^6+36}$
10. $\displaystyle\sum_{n=1}^{\infty} \frac{4n^3}{n^8+64}$