11.2 Series & Partial Sums

1. $S_n = 3 - \frac{1}{n}$.

$$\lim_{n\to\infty} S_n = 3 - \lim_{n\to\infty} \frac{1}{n} = 3.$$

2. $S_n = \frac{5n}{n+1}$.

$$S_n = \frac{5n}{n\left(1+\frac{1}{n}\right)} = \frac{5}{1+\frac{1}{n}} \longrightarrow 5.$$

3. $S_n = \frac{2n^2+1}{n^2+2n+1}$.

$$S_n = \frac{n^2\left(2+\frac{1}{n^2}\right)}{n^2\left(1+\frac{2}{n}+\frac{1}{n^2}\right)} = \frac{2+\frac{1}{n^2}}{1+\frac{2}{n}+\frac{1}{n^2}} \longrightarrow 2.$$

4. $S_n = \frac{4n^2}{n^2+1}$.

$$S_n = \frac{4}{1+\frac{1}{n^2}} \longrightarrow 4.$$

5. $S_n = 1 - \frac{2}{n^2+1}$.

$$\lim_{n\to\infty} S_n = 1 - 2\lim_{n\to\infty} \frac{1}{n^2+1} = 1.$$

6. $S_n = \frac{6n^2+3n+2}{2n^2+5n+1}$.

$$S_n = \frac{6+\frac{3}{n}+\frac{2}{n^2}}{2+\frac{5}{n}+\frac{1}{n^2}} \longrightarrow \frac{6}{2} = 3.$$

7. $S_n = \frac{5n}{\ln(n+1)+2n}$.

$$S_n = \frac{5}{\frac{\ln(n+1)}{n}+2}.$$

Since $\frac{\ln(n+1)}{n} \to 0$, it follows that

$$\lim_{n\to\infty} S_n = \frac{5}{2}.$$

8. $S_n = \frac{n+4}{\sqrt{n^2+9}}$.

$$S_n = \frac{n\left(1+\frac{4}{n}\right)}{n\sqrt{1+\frac{9}{n^2}}} = \frac{1+\frac{4}{n}}{\sqrt{1+\frac{9}{n^2}}} \longrightarrow 1.$$

9. $S_n = \frac{4n+\ln n}{3n+1}$.

$$S_n = \frac{4+\frac{\ln n}{n}}{3+\frac{1}{n}}.$$

Since $\frac{\ln n}{n} \to 0$, we obtain

$$\lim_{n\to\infty} S_n = \frac{4}{3}.$$

10. $S_n = \dfrac{n^2+1}{n\ln n + 1}$.

$$S_n = \frac{n^2\left(1+\frac{1}{n^2}\right)}{n\ln n\left(1+\frac{1}{n\ln n}\right)} = \frac{n}{\ln n}\cdot\frac{1+\frac{1}{n^2}}{1+\frac{1}{n\ln n}}.$$

The second factor tends to 1, while $\dfrac{n}{\ln n} \to \infty$. Hence

$$\lim_{n\to\infty} S_n = \infty,$$

so the series diverges.