Taylor Polynomials

A **Taylor polynomial** is a polynomial that approximates a function near a chosen center $x = a$. The n-th degree Taylor polynomial uses the value of the function and its first n derivatives at a:

$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

Near $x = a$, the graph of $T_n(x)$ closely matches the graph of $f(x)$. In general, higher-degree Taylor polynomials give better approximations near the center.

Find the degree 4 Taylor polynomial for $f(x)$ centered at the given value of a.

1. $f(x) = x^2 e^x, \quad a = 0$
2. $f(x) = \dfrac{1}{1+x^2}, \quad a = 1$
3. $f(x) = \cos(x^2), \quad a = 0$
4. $f(x) = \sin(2x), \quad a = \dfrac{\pi}{4}$
5. $f(x) = \ln(1+2x), \quad a = 0$
6. $f(x) = \tan^{-1}(x), \quad a = 0$
7. $f(x) = \dfrac{x}{1-x}, \quad a = 0$
8. $f(x) = e^{x^2}, \quad a = 0$
9. $f(x) = \dfrac{1}{\sqrt{1-x}}, \quad a = 0$