10.2 Calculus with Parametric Curves

Theorem. Let $x = f(t)$ and $y = g(t)$ be a parametric curve, where f and g are differentiable functions of t. If y is a differentiable function of x, the derivative $\frac{dy}{dx}$ is given by

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad \text{provided that } \frac{dx}{dt} \neq 0.$$

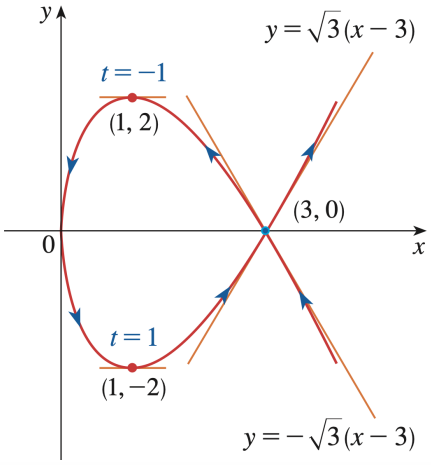
Proof.

□

Example. A curve C is defined by the parametric equations $x = t^2, y = t^3 - 3t$.

(a) Show that C has two tangents at the point $(3, 0)$ and find their equations.

(b) Find the points on C where the tangent is horizontal or vertical.



Example.

(a) Find the tangent to the cycloid $x = r(\theta - \sin\theta), y = r(1 - \cos\theta)$ at the point where $\theta = \pi/3$.

(b) At what points is the tangent horizontal? When is it vertical?