10.2 Tangents to Parametric Curves

Suppose $x = f(t)$ and $y = g(t)$ are differentiable functions. If $\frac{dx}{dt} \neq 0$, then the slope of the curve is:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}.$$

To find the equation of the tangent line at a specific parameter value $t = t_0$:

- Find the point on the curve: $x_0 = f(t_0)$ and $y_0 = g(t_0)$.
- Calculate the slope at that point: $m = \left.\frac{dy}{dx}\right|_{t=t_0}$.
- Use the point-slope form:

$$y - y_0 = m(x - x_0).$$

From the slope formula, we can determine the orientation of the tangents:

- **Horizontal Tangent:** Occurs when $\frac{dy}{dx} = 0$. This happens if $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$.
- **Vertical Tangent:** Occurs when $\frac{dy}{dx}$ is undefined. This happens if $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$.

Tangents to Parametric Curves Problems

1. Given the parametric equations:

$$x = t^2 + 1, \quad y = 2t - 3,$$

find the slope of the curve at $t = 1$.

2. Consider the curve defined by:

$$x = \sin t, \quad y = \cos t, \quad 0 \leq t \leq 2\pi.$$

Find the points where the curve has a horizontal tangent.

3. Find the equation of the tangent line to the curve:

$$x = e^t, \quad y = e^{-t}$$

at $t = 0$.

4. Determine the slope of the tangent line for the parametric curve:

$$x = t - \sin t, \quad y = 1 - \cos t$$

at $t = \frac{\pi}{4}$.

5. Compute the equation of the tangent line to the curve:

$$x = \ln(t), \quad y = t^2$$

at $t = 1$.

6. Given the parametric equations:

$$x = 3t^2 + 2, \quad y = 4t^3 - 5,$$

find the slope of the tangent line at $t = -1$.

7. Find the equation of the tangent line to the parametric curve:

$$x = t^2 + 2t, \quad y = 3t - 1$$

at the point corresponding to $t = 2$.