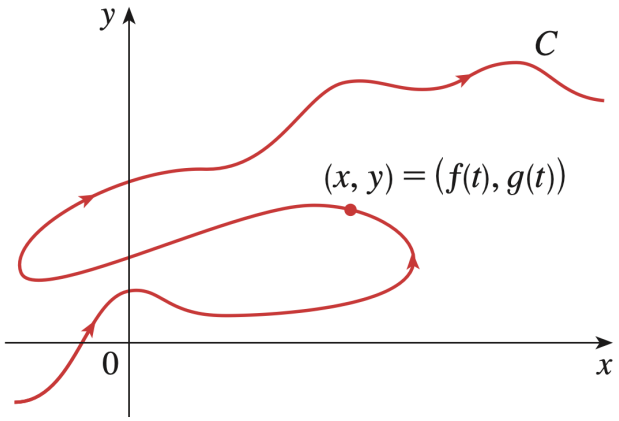
10.1 Curves Defined by Parametric Equations

Question. Imagine that a particle moves along the curve C shown below.



Is it possible to describe C by an equation of the form $y = f(x)$?

Question. How can we describe the curve C?

Remark. The parameter t does not necessarily represent time. However, in many applications of parametric curves, t does denote time and we can interpret $(x, y) = (f(t), g(t))$ as the position of a moving object at time t.

Example. Sketch and identify the curve defined by the parametric equations

$$x = t^2 - 2t \qquad y = t + 1$$

Remark. The process used in this example is called eliminating the parameter.

Remark. Eliminating the parameter can be helpful in identifying the shape of the parametric curve, but we lose some information in the process. The equation in x and y describes the curve the particle travels along, whereas the parametric equations could tell us where the particle is at any given time and indicate the direction of motion.

Remark. It is not always possible to eliminate the parameter from parametric equations. There are many parametric curves that don't have a representation as an equation in x and y.

Example. Sometimes we restrict t to lie in a particular interval. Graph the parametric curve given by

$$x = t^2 - 2t \qquad y = t + 1 \qquad 0 \leq t \leq 4$$

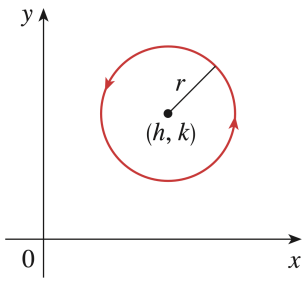
Example. What curve is represented by the following parametric equations?

$$x = \cos t \qquad y = \sin t \qquad 0 \leq t \leq 2\pi$$

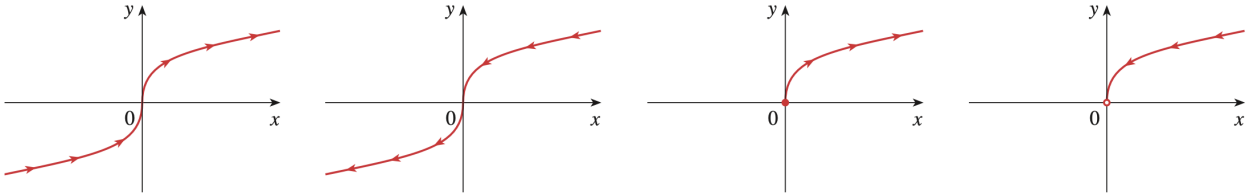
Example. What curve is represented by the given parametric equations?

$$x = \sin 2t \qquad y = \cos 2t \qquad 0 \leq t \leq 2\pi$$

Example. Find parametric equations for the circle with center (h, k) and radius r.



Example. Each of the following sets of parametric equations gives the position of a moving particle at time t.



(a) $x = t^3, y = t$ (b) $x = -t^3, y = -t$ (c) $x = t^{3/2}, y = \sqrt{t}$ (d) $x = e^{-3t}, y = e^{-t}$

In each case, eliminating the parameter gives $x = y^3$, so each particle moves along the cubic curve $x = y^3$; however, the particles move in different ways

Example. Sketch the curve with parametric equations $x = \sin t, y = \sin^2 t$.

Example. Consider the curve represented by the parametric equations

$$x = t^2 + 1, \quad y = t^3 - 2t \quad \text{for} \quad -2 \leq t \leq 2.$$

Find the point at which the curve intersects itself and the corresponding values of t.