10.1 Parametric Curves

Definition (Parametric Equations)**.** A curve in the plane can be described by a pair of parametric equations

$$x = f(t), \quad y = g(t),$$

where t is a parameter (often representing time). The set of all points $\big(f(t), g(t)\big)$ traced out as t varies over an interval $[a, b]$ is called a *parametric curve.*

Eliminating the Parameter: Sometimes we can solve one of the equations (e.g. $x = f(t)$) for t and substitute into the other ($y = g(t)$) to obtain a Cartesian equation $F(x, y) = 0$. This process is called *eliminating the parameter*. However, doing so can lose information about direction and speed.

Remark. Different parametric equations can represent the same geometric curve. However, the direction of traversal and the speed at which the curve is traced depend on how x and y change with respect to the parameter t.

Parametric Curves Problems

1. Sketch and identify the curve.

$$x = t^2 - 4t, \qquad y = t + 2, \qquad 0 \le t \le 5.$$

2. Sketch and identify the curve.

$$x = 2\cos t - 1, \qquad y = 2\sin t + 3, \qquad 0 \le t \le 2\pi.$$

3. Sketch and identify the curve.

$$x = \sin(2t), \qquad y = \cos(2t), \qquad 0 \le t \le 2\pi.$$

4. Sketch and identify the curve.

$$x = t^3, \qquad y = t, \qquad -2 \le t \le 2.$$

5. Sketch and identify the curve.

$$x = \sin t, \qquad y = \sin^2 t, \qquad 0 \le t \le 2\pi.$$

6. Find a parametrization $\mathbf{r}(t) = (x(t), y(t))$ of the line segment from

$$P_0 = (-2, 1) \quad \text{to} \quad P_1 = (4, -3),$$

   using $0 \le t \le 1$.

7. Give parametric equations for the circle of radius 3 centered at $(2, -1)$ that
   - starts at the *rightmost* point of the circle, and
   - travels *clockwise* exactly once.

8. Give a parametrization of the upper semicircle

$$x^2 + y^2 = 9, \qquad y \ge 0,$$

   traced from $(-3, 0)$ to $(3, 0)$ (left to right). State an interval for t.