Calculus I Review for Success in Calculus II

Contents

1 Algebra and Trigonometry

In Calculus II, the new ideas are mostly *integration techniques*. What makes those techniques feel hard is usually not the calculus—it is the algebra and trigonometry needed to *rewrite* an integrand into a usable form. If these skills are slow or error-prone, every method in Calc II becomes unnecessarily painful.

1.1 Core algebra patterns

- **Factoring identities.**

  $$a^2 - b^2 = (a-b)(a+b), \qquad a^3 - b^3 = (a-b)(a^2+ab+b^2), \qquad a^3 + b^3 = (a+b)(a^2-ab+b^2).$$

- **Completing the square.** Rewrite a quadratic so it has the form $(\cdot)^2 + \text{constant}$. First factor out a, then add and subtract the term that makes a perfect square:

  $$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x\right) + c = a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right).$$

  In particular, $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$.

- **Rationalizing.** Multiply by the conjugate:

  $$\sqrt{u} - \sqrt{v} \quad \rightsquigarrow \quad (\sqrt{u} - \sqrt{v})\frac{\sqrt{u}+\sqrt{v}}{\sqrt{u}+\sqrt{v}} = \frac{u-v}{\sqrt{u}+\sqrt{v}}.$$

- **Solving for a substitution variable.** In Calc II, you will repeatedly solve equations like $u = g(x)$ for x and rewrite other expressions in terms of u. Practice doing this cleanly (and tracking domain restrictions when needed).

- **Partial fraction setup.** When a rational function appears, factor the denominator as completely as possible over $\mathbb{R}$ before decomposing. (Calc II will develop the decomposition rules; the prerequisite is strong factoring.)

1.2 Exponent and logarithm laws

Exponent laws (for $a > 0$; when relevant assume $a \neq 1$):

$$a^m a^n = a^{m+n}, \qquad \frac{a^m}{a^n} = a^{m-n}, \qquad (a^m)^n = a^{mn}, \qquad (ab)^n = a^n b^n, \qquad a^{-n} = \frac{1}{a^n}.$$

Logarithm laws (for $x > 0$, $a > 0$, $a \neq 1$):

$$\ln(ab) = \ln a + \ln b, \qquad \ln\left(\frac{a}{b}\right) = \ln a - \ln b, \qquad \ln(a^r) = r\ln a, \qquad e^{\ln x} = x, \qquad \ln(e^x) = x.$$

Common pitfall. The identity $\ln(a+b) = \ln a + \ln b$ is *false*. Log rules apply to products, quotients, and powers—not sums.

1.3 Trigonometry essentials

Pythagorean identities (memorize):

$$\sin^2 x + \cos^2 x = 1, \qquad 1 + \tan^2 x = \sec^2 x, \qquad 1 + \cot^2 x = \csc^2 x.$$

Double-angle identities (used constantly in trig integrals):

$$\sin(2x) = 2\sin x \cos x, \qquad \cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1.$$

Calc II connection. Most trig-integration "tricks" are just identity rewrites that convert an integrand into a basic antiderivative form.

Key unit-circle values (first quadrant)

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

To extend beyond the first quadrant, use reference angles and the signs of sin, cos, and tan by quadrant. In Calc II, the most common errors are incorrect signs after a trig substitution or identity rewrite.

2 Limits and Continuity

What a limit says. $\lim_{x \to c} f(x) = L$ means that when x is taken *close* to c (not necessarily equal to c), the values $f(x)$ are forced *close* to L. Limits describe *local behavior* near $x = c$, not the value of the function at c.

2.1 Computing limits: the standard toolkit

- **Limit laws.** If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist, then

$$\lim_{x\to c}(f \pm g) = \lim_{x\to c} f \pm \lim_{x\to c} g, \qquad \lim_{x\to c}(fg) = \left(\lim_{x\to c} f\right)\left(\lim_{x\to c} g\right),$$

$$\lim_{x\to c} \frac{f}{g} = \frac{\lim_{x\to c} f}{\lim_{x\to c} g} \quad \text{provided} \quad \lim_{x\to c} g \neq 0.$$

- **Continuity shortcut.** If f is continuous at c, then

$$\lim_{x\to c} f(x) = f(c).$$

  Polynomials are continuous everywhere; rational, logarithmic, and trigonometric functions are continuous on their domains; compositions of continuous functions are continuous where defined.

- **Indeterminate forms signal "rewrite."** If direct substitution produces $0/0$ or ∞/∞, do algebra before anything else:
  - *Factor and cancel* common factors.
  - *Combine fractions* to create a single rational expression.
  - *Rationalize* to remove radicals in a numerator/denominator.
  - *Use trig identities* to expose a cancelable factor.
  - For rational functions as $x \to \pm\infty$: *divide by the highest power of* x.

- **One-sided limits.** When a function behaves differently from the left and right, compute $\lim_{x\to c^-} f(x)$ and $\lim_{x\to c^+} f(x)$. The two-sided limit exists only if they match.

2.2 Continuity and types of discontinuity

A function f is continuous at $x = c$ if:

$$\lim_{x\to c} f(x) \text{ exists}, \qquad f(c) \text{ is defined}, \qquad \lim_{x\to c} f(x) = f(c).$$

Common discontinuities:

- **Removable:** $\lim_{x\to c} f(x)$ exists, but $f(c)$ is missing or not equal to the limit (a "hole").
- **Jump:** $\lim_{x\to c^-} f(x)$ and $\lim_{x\to c^+} f(x)$ exist but are different.
- **Infinite:** $f(x) \to \pm\infty$ as $x \to c$ (vertical asymptote).

2.3 Intermediate Value Theorem (IVT)

If f is continuous on $[a, b]$, then f takes every value between $f(a)$ and $f(b)$. In particular, if $f(a)$ and $f(b)$ have opposite signs, then there exists $c \in (a, b)$ such that $f(c) = 0$.

Calc II connection. IVT is the clean justification for the existence of solutions (roots/intersections) without finding them exactly. This perspective becomes important when exact antiderivatives do not exist and numerical/approximate thinking matters.

3 Derivatives

Derivatives measure *instantaneous change*. Even in Calculus II (an integration-heavy course), derivatives appear constantly: they drive substitution, support approximation and error estimates, and connect rates, accumulation, and geometry.

3.1 Definition and meaning

Definition.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Interpretations. $f'(x)$ is the slope of the tangent line to $y = f(x)$ at x, and the instantaneous rate of change of f with respect to x.

Tangent line at $x = a$.

$$y = f(a) + f'(a)(x - a).$$

3.2 Differentiation rules

Linearity.

$$(cf)' = cf', \qquad (f \pm g)' = f' \pm g'.$$

Power rule.

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}.$$

Product and quotient rules.

$$(uv)' = u'v + uv', \qquad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}.$$

Chain rule.

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x).$$

Common pitfall (chain rule). The most frequent error is forgetting the factor $g'(x)$. When differentiating a composition, write the inside derivative explicitly before simplifying.

3.3 Derivative table

Function	**Derivative**
c	0
x^n	nx^{n-1}
$e^x,\ a^x$	$e^x,\ a^x \ln a$
$\ln x,\ \log_a x$	$\frac{1}{x},\ \frac{1}{x \ln a}$
$\sin x,\ \cos x$	$\cos x,\ -\sin x$
$\tan x,\ \cot x$	$\sec^2 x,\ -\csc^2 x$
$\sec x,\ \csc x$	$\sec x \tan x,\ -\csc x \cot x$
$\arcsin x,\ \arccos x$	$\frac{1}{\sqrt{1-x^2}},\ -\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

3.4 Implicit differentiation

Many relationships are given implicitly (an equation linking x and y without solving for y). Differentiate both sides with respect to x, treating y as a function $y(x)$.

Key pattern. If $y = y(x)$, then

$$\frac{d}{dx}\left[y^n\right] = ny^{n-1}y', \qquad \frac{d}{dx}\left[\sin(y)\right] = \cos(y)\, y', \qquad \frac{d}{dx}\left[e^y\right] = e^y\, y'.$$

Rule of thumb. Whenever you differentiate an expression containing y, a factor of y' appears.

3.5 Logarithmic differentiation

Use logarithmic differentiation when variables appear in both the base and exponent, or when products/quotients/powers simplify after taking logs (e.g. $y = x^x$, $y = (\sin x)^{\sqrt{x}}$).

Workflow.

1. Start with $y =$ (expression).
2. Take ln of both sides: $\ln y = \ln(\text{expression})$.
3. Use log laws to simplify.
4. Differentiate both sides (remember $\frac{d}{dx}[\ln y] = \frac{y'}{y}$).
5. Solve for y' and substitute y back in if needed.

3.6 Linearization and differentials

Linear approximation at $x = a$.

$$f(x) \approx f(a) + f'(a)(x - a).$$

Differentials.

$$dy = f'(x)\,dx \quad \text{and} \quad \Delta y \approx dy$$

provide fast error estimates for small changes dx.

Calc II connection. Linearization and differentials are the conceptual starting point for many approximation and error-bound ideas you will see again (sometimes implicitly) in Calc II.

4 Derivative Applications You Need in Calc II

Calculus II is not "derivative-free." You will repeatedly use derivative-based reasoning to justify substitutions, analyze accumulation functions, estimate error, and interpret physical quantities. These applications are the language of calculus.

4.1 Critical points, extrema, and optimization

Critical numbers. A number c in the domain of f is a *critical number* if

$$f'(c) = 0 \quad \text{or} \quad f'(c) \text{ does not exist.}$$

Key idea. Absolute maxima/minima can occur at *endpoints* or at *critical numbers.*

- **Absolute extrema on a closed interval.** To find the absolute maximum/minimum of f on $[a, b]$:
  1. Find all critical numbers in (a, b).
  2. Evaluate f at those points and at the endpoints a and b.
  3. Compare the function values; the largest is the absolute maximum and the smallest is the absolute minimum.
- **Optimization workflow.**
  1. *Define variables* with a diagram when appropriate.
  2. *Write the objective function* $Q =$ quantity to maximize/minimize.
  3. *Use constraints* to rewrite Q in *one variable.*
  4. Differentiate: compute $Q'(x)$ and solve $Q'(x) = 0$ (and check where Q' does not exist).
  5. *Justify the optimum* (endpoint check, first/second derivative test, or sign analysis).
  6. *Interpret the result* with units and a complete sentence.

4.2 Monotonicity and concavity (first and second derivatives)

Increasing/decreasing. If $f'(x) > 0$ on an interval, then f is increasing there. If $f'(x) < 0$, then f is decreasing.

Concavity. If $f''(x) > 0$ on an interval, then f is concave up there. If $f''(x) < 0$, then f is concave down.

- **Inflection points.** A point $x = c$ is an inflection point candidate if $f''(c) = 0$ or $f''(c)$ does not exist. It is an inflection point *only if* concavity changes sign across c.
- **Local extrema tests.**

- *First derivative test:* if f' changes from + to −, you have a local maximum; if − to +, a local minimum.
- *Second derivative test:* if $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c; if $f''(c) < 0$, a local maximum (inconclusive if $f''(c) = 0$).

4.3 Mean Value Theorem (MVT)

Mean Value Theorem. If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Interpretation. At some point, the instantaneous rate of change equals the average rate of change over the whole interval.

Calc II connection. MVT underlies error estimates and approximation statements (for example, when bounding the error of a linear approximation or when reasoning about accumulation functions). Even when not named explicitly, its logic appears frequently.

4.4 Motion: position, velocity, acceleration

Let $s(t)$ be position.

$$v(t) = s'(t) \quad \text{(velocity)}, \qquad a(t) = v'(t) = s''(t) \quad \text{(acceleration)}.$$

Speed is $|v(t)|$, not $v(t)$.

- **Direction vs. magnitude.** The sign of $v(t)$ gives direction of motion; $|v(t)|$ gives how fast.
- **Speeding up / slowing down.** A particle speeds up when velocity and acceleration have the same sign ($v \cdot a > 0$), and slows down when they have opposite signs ($v \cdot a < 0$).
- **Units.** If s is in meters and t is in seconds, then v is m/s and a is m/s^2. Unit discipline prevents many "answer looks right" mistakes.

5 Integrals (Calc I Foundations That Calc II Builds On)

A definite integral measures *accumulation*. In Calculus II, you will learn many new techniques for *computing* integrals, but the meaning of the integral and the Fundamental Theorem of Calculus remain the organizing principles.

5.1 What a definite integral is

$$\int_a^b f(x)\,dx$$

is the **net signed area** under $y = f(x)$ from $x = a$ to $x = b$: area above the x-axis counts positive, and area below counts negative.

Language you must be fluent in.

- *Net area* (signed): $\int_a^b f(x)\,dx$.
- *Total area* (geometric): $\int_a^b |f(x)|\,dx$.

In Calc II, problems often switch between these, and the difference matters.

5.2 Riemann sums

Definition via Riemann sums. If f is integrable on $[a, b]$, then

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x, \qquad \Delta x = \frac{b-a}{n},$$

where x_i^* is any sample point in the i-th subinterval.

You should be able to:

- compute $\Delta x = \frac{b-a}{n}$,
- write $x_i = a + i\Delta x$ (right endpoints) or $x_i = a + (i-1)\Delta x$ (left endpoints),
- convert between a definite integral and a Riemann sum expression.

5.3 Fundamental Theorem of Calculus (FTC)

FTC I (evaluation). If $F'(x) = f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

FTC II (accumulation derivatives). If $A(x) = \int_a^x f(t)\,dt$, then

$$A'(x) = f(x).$$

Chain rule version. If $A(x) = \int_a^{g(x)} f(t)\,dt$, then

$$A'(x) = f(g(x))\,g'(x).$$

5.4 Integral properties you constantly use

- **Linearity:** $\int_a^b \big(c_1 f(x) + c_2 g(x)\big)\,dx = c_1 \int_a^b f(x)\,dx + c_2 \int_a^b g(x)\,dx.$
- **Additivity over intervals:** $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$
- **Reversing limits:** $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$

5.5 Antiderivative table

Integrand	Antiderivative
k	$kx + C$
$x^n \quad (n \neq -1)$	$\dfrac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
$e^x,\ a^x$	$e^x + C,\ \dfrac{a^x}{\ln a} + C$
$\sin x,\ \cos x$	$-\cos x + C,\ \sin x + C$
$\sec^2 x,\ \csc^2 x$	$\tan x + C,\ -\cot x + C$
$\sec x \tan x,\ \csc x \cot x$	$\sec x + C,\ -\csc x + C$
$\frac{1}{1+x^2}$	$\arctan x + C$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x + C$

Calc II connection. New integration techniques are mostly methods for rewriting an integrand until it matches one of these antiderivative patterns (possibly after a substitution). If you do not instantly recognize the forms in this table, the rest of Calc II slows down dramatically.

6 What Calc II Will Add (and the Calc I Skill Each Topic Uses)

Orientation. Calculus II expands the idea of *accumulation* in three directions:

- **New integration techniques** (so more functions become integrable),
- **New applications of definite integrals** (geometry and physical quantities),
- **Sequences and series** (infinite processes with rigorous control).

The content is very learnable, but only if the Calc I fundamentals are fast and reliable.

Skill map: Calc II topics and the Calc I foundations they require

Calc II topic	**Calc I prerequisite it relies on**
u-substitution	Chain rule fluency; solving for a substitution; algebra/trig simplification
Integration by parts	Product rule *in reverse*; clean algebra; antiderivative recognition
Trig integrals	Identities; unit circle values; algebraic rewriting
Trig substitution	Pythagorean identities; right-triangle geometry; careful back-substitution
Partial fractions	Factoring; rational expression algebra; equation solving
Improper integrals	Limits at ∞ and at discontinuities; comparison intuition
Applications (area/volume/arc length/surface area)	Definite integrals; function modeling; units; geometric interpretation
Parametric and polar calculus	Chain rule; FTC; comfort with alternate variables and notation
Sequences and series	Limit skills; algebra; precise notation; handling indeterminate forms

If you are struggling in Calc II, the fix is often *not* more Calc II practice. It is usually one of:

- weak algebra/trig rewrites,
- missing chain rule factors,
- not recognizing derivative/antiderivative patterns quickly,
- shaky limit reasoning (especially near ∞ or vertical asymptotes).

7 Readiness Check (No Solutions)

Readiness check. If you can complete the problems below *quickly and correctly* (with clean work and minimal algebra errors), you have the Calc I foundation needed to succeed in Calc II.

Problems

1. Compute $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$.
2. Compute $\lim_{x\to 0} \dfrac{\sin(5x)}{x}$.
3. Find the equation of the tangent line to $y=\sqrt{x}$ at $x=4$.
4. Differentiate: $y=(x^2+1)^7$.
5. Differentiate: $y=\dfrac{x\sin x}{e^x}$.
6. Implicitly differentiate: $x^2+y^2=25$ and solve for y'.
7. Log-differentiate: $y=x^{\sin x}$.
8. Evaluate: $\displaystyle\int_0^2 (3x^2-4)\,dx$.
9. Differentiate: $\dfrac{d}{dx}\left(\displaystyle\int_1^x \frac{\ln t}{t}\,dt\right)$.
10. Find the absolute maximum of $f(x)=x(4-x)$ on $[0,4]$.

Self-grading checklist

- Did you simplify algebra and trig expressions correctly (no sign mistakes, no dropped factors)?
- Did every chain rule step include the derivative of the inside function?
- For implicit/log differentiation, did you correctly solve for y' and present a clean final expression?
- For the FTC derivative, did you apply the "upper limit derivative" correctly (and include $g'(x)$ when needed)?
- For optimization, did you check both endpoints and interior critical points on $[0,4]$?