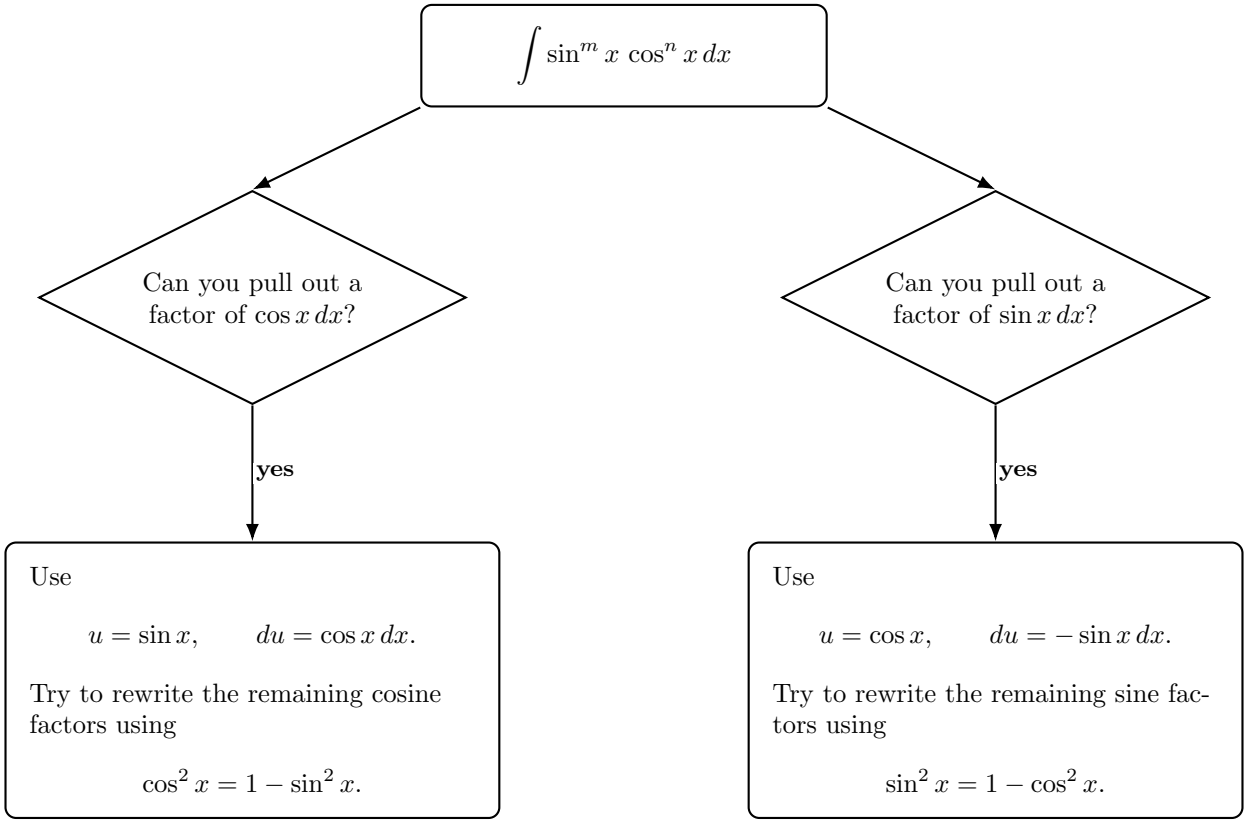
7.2 Trigonometric Integrals

Integrals of Powers of Sine and Cosine

We begin with integrals built from powers of sine and cosine:

$$\int \sin^m x \, \cos^n x \, dx.$$

These problems are u-substitution problems disguised with trigonometry. Our strategy is to pull out a derivative du, then use trig identities to rewrite the leftover factors in terms of the chosen u.



If neither substitution works cleanly, use power-reduction identities:

$$\sin^2 x = \frac{1 - \cos(2x)}{2}, \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}.$$

Then simplify and integrate term-by-term. Useful identity:

$$\sin x \cos x = \frac{1}{2} \sin(2x).$$

Example. Find $\int \sin^5 x \cos^2 x \, dx$.

Example. Find $\int \sin^4 x \, dx$.

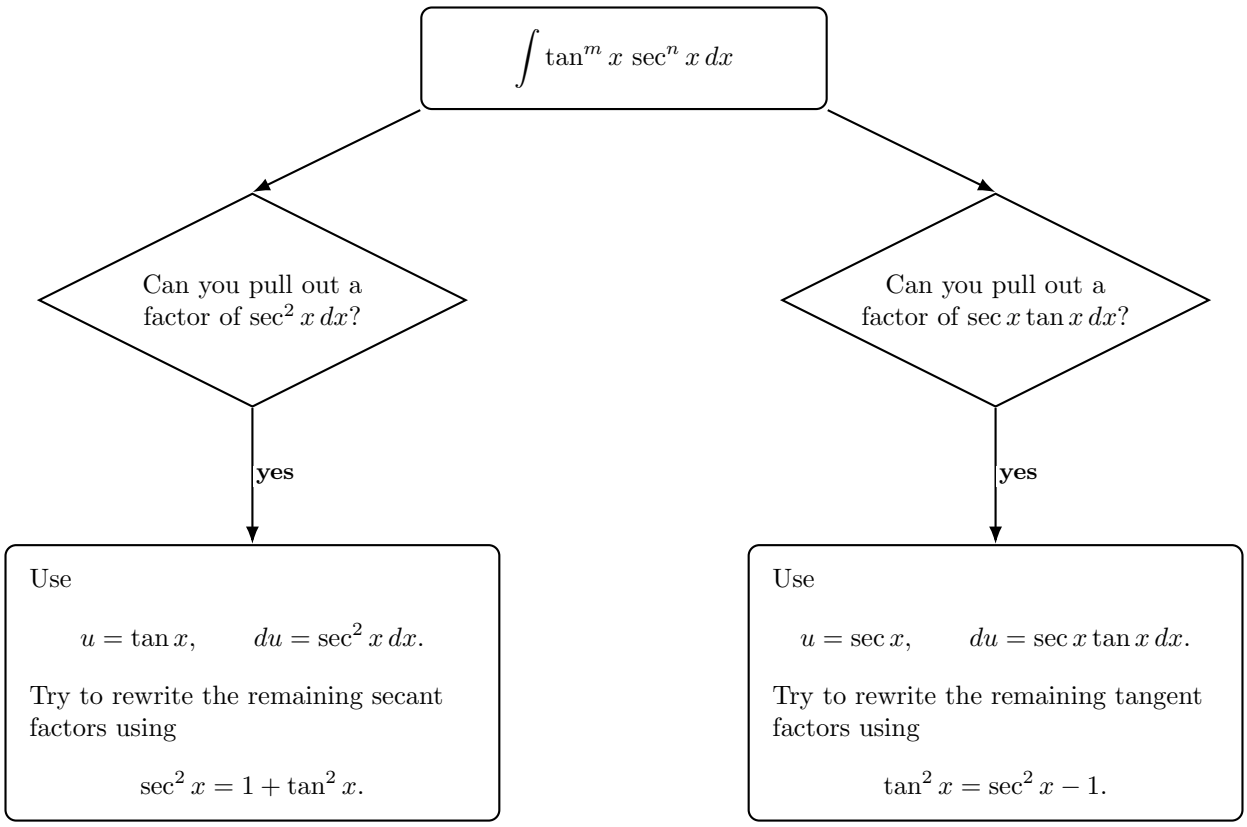
Integrals of Powers of Secant and Tangent

Now we consider integrals built from powers of tangent and secant:

$$\int \tan^m x \, \sec^n x \, dx.$$

Once again, our strategy is to pull out a derivative du, then use trig identities to rewrite the leftover factors in terms of the chosen u. The key derivatives are

$$\frac{d}{dx}(\tan x) = \sec^2 x, \qquad \frac{d}{dx}(\sec x) = \sec x \tan x.$$



If neither substitution works cleanly, try to rewrite in terms of $\sin x$ and $\cos x$

$$\tan x = \frac{\sin x}{\cos x}, \qquad \sec x = \frac{1}{\cos x},$$

then try $u = \sin x$ or $u = \cos x$. Alternatively, convert everything to $\sec x$ and use integration by parts.

Example. Evaluate $\int \tan^6 x \sec^4 x \, dx$.

Example. Find $\int \tan^5\theta \sec^7\theta \, d\theta$.

Remark. In some cases, the guidelines for integrating powers of $\tan x$ and $\sec x$ are not as straightforward. We may need to use trigonometric identities, integration by parts, or creative problem-solving techniques. Two important integrals to remember for these cases are:

$$\int \tan x \, dx = \ln|\sec x| + C$$

and

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C.$$

Example. Find $\int \tan^3 x \, dx$.

Example. Find $\int \sec^3 x \, dx$.