6.4 Work

Definition. In physics, a **force** is defined as a push or pull on an object. For example, a horizontal push of a book across a table, or the downward pull of gravity on a ball. Mathematically, if an object moves along a straight line with position function $s(t)$, the force F acting on the object is given by Newton's Second Law of Motion:

$$F = ma = m\frac{d^2s}{dt^2},$$

where m is the mass of the object, and a is the acceleration of the object.

Definition. Work is defined as the product of the force F acting on an object and the distance d the object moves:

$$W = Fd \quad (\text{work} = \text{force} \times \text{distance}).$$

- If F is measured in newtons (N) and d in meters (m), W is measured in *joules* (J):

$$1 \text{ J} = 1 \text{ N} \cdot \text{m}.$$

- If F is measured in pounds (lb) and d in feet (ft), W is measured in *foot-pounds* (ft-lb):

$$1 \text{ ft-lb} \approx 1.36 \text{ J}.$$

Example.

(a) How much work is done in lifting a 1.2-kg book off the floor to put it on a desk that is 0.7 m high? Use the fact that the acceleration due to gravity is $g = 9.8$ m/s^2.

(b) How much work is done in lifting a 20-lb weight 6 ft off the ground?

When the force is constant, work is simply the product of force and distance: $W = F \cdot d$. If the force varies with position, we treat it as a function $F(x)$ defined on the interval $[a, b]$. How can we compute the work done as an object moves from $x = a$ to $x = b$?

- Approximate the work done over a small subinterval:

- Approximate the total work over the entire interval $[a, b]$:

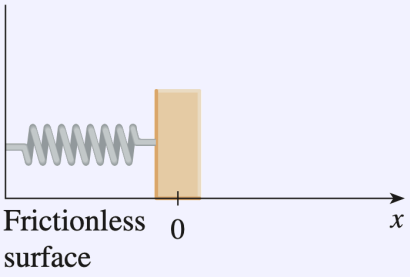
- Calculate the exact total work for a variable force:

Example. When a particle is located a distance x feet from the origin, a force of $x^2 + 2x$ pounds acts on it. How much work is done in moving it from $x = 1$ to $x = 3$?

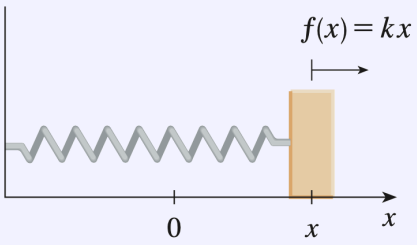
Theorem (Hooke's Law)**.** The force required to maintain a spring stretched x units beyond its natural length is proportional to x:

$$f(x) = kx$$

where k is a positive constant called the spring constant. Hooke's Law holds provided that x is not too large.



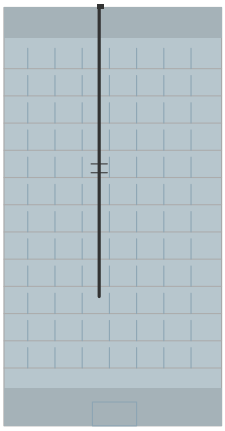
(a) Natural position of spring



(b) Stretched position of spring

Example. A force of 40 N is required to hold a spring that has been stretched from its natural length of 10 cm to a length of 15 cm. How much work is done in stretching the spring from 15 cm to 18 cm?

Example. A 200-lb cable is 100 ft long and hangs vertically from the top of a tall building.



(a) How much work is required to lift the cable to the top of the building?

(b) How much work is required to pull up only 20 feet of the cable?

Example. A tank has the shape of an inverted circular cone with a height of 10 m and a base radius of 4 m. It is filled with water to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000 kg/m^3.)