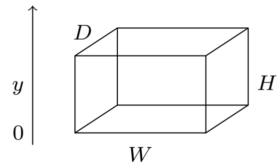
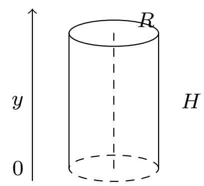
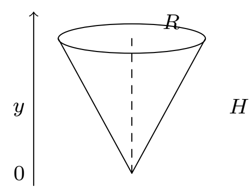
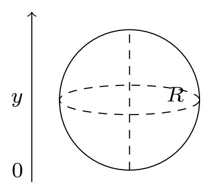
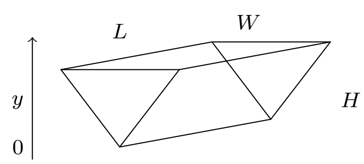
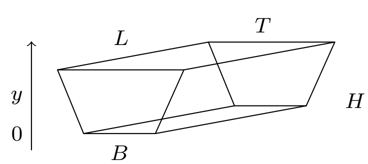
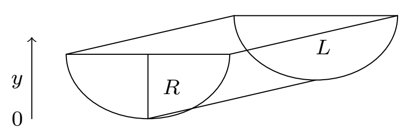
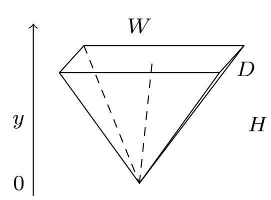
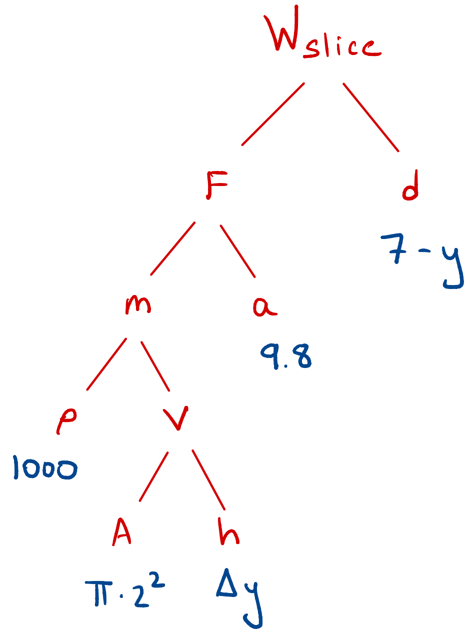
Common Tank Shapes

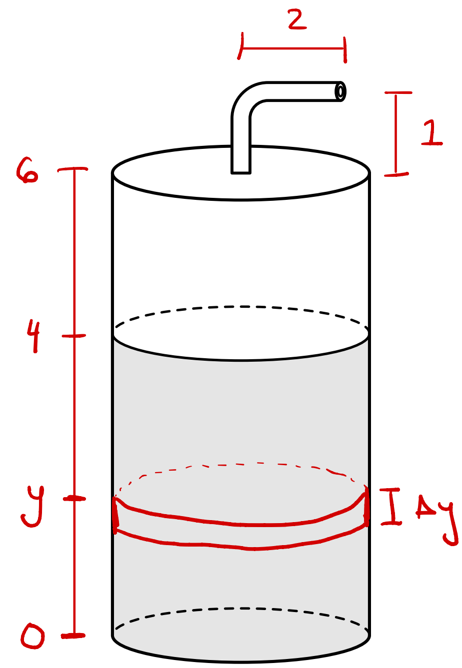
Heights are measured from the bottom of the tank, so the horizontal slice is taken at height y.

Do NOT memorize these, just know how to find them

<table>
<tr><th>Tank</th><th>Diagram</th><th>Slice Area</th></tr>
<tr><td>Rectangular Box Tank</td><td></td><td>$A(y) = WD$</td></tr>
<tr><td>Vertical Cylinder</td><td></td><td>$A(y) = \pi R^2$</td></tr>
<tr><td>Conical Tank</td><td></td><td>$A(y) = \pi \left(\frac{R}{H} y \right)^2$</td></tr>
<tr><td>Spherical Tank</td><td></td><td>$A(y) = \pi \left(R^2 - (y - R)^2 \right)$</td></tr>
<tr><td>Triangular Trough</td><td></td><td>$A(y) = L \left(\frac{W}{H} y \right)$</td></tr>
<tr><td>Trapezoidal Trough</td><td></td><td>$A(y) = L \left(B + \frac{T - B}{H} y \right)$</td></tr>
<tr><td>Semi-Cylindrical Trough</td><td></td><td>$A(y) = 2L\sqrt{R^2 - (R - y)^2}$</td></tr>
<tr><td>Pyramidal Tank</td><td></td><td>$A(y) = WD \left(\frac{y}{H} \right)^2$</td></tr>
</table>

Problem. A vertical cylindrical tank has radius 2 m and height 6 m. The tank contains water to a depth of 4 m. A spout extends 1 m above the top of the tank. Using $\rho = 1000$ kg/m^3 and $g = 9.8$ m/s^2, set up an integral for the work required to pump all the water out through the spout.





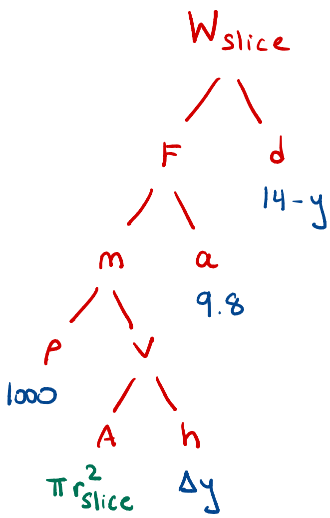
$$W_{slice} = 1000 \cdot 9.8 \cdot 4\pi \cdot (7-y)\,\Delta y$$

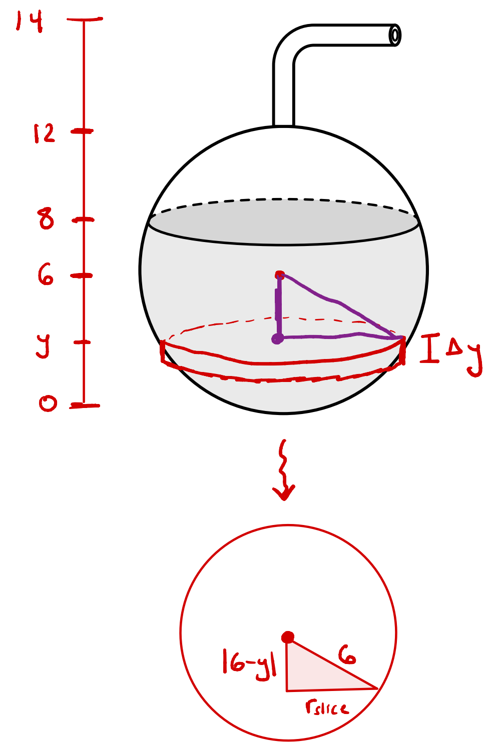
$$W_{Total} = \int_0^4 1000 \cdot 9.8 \cdot 4\pi\,(7-y)\,dy$$

The slices are between 0 and 4

Homework

Problem. A spherical tank has radius 6 m. The tank is filled with water to a depth of 8 m, measured from the bottom. A spout extends 2 m above the top of the tank. Using $\rho = 1000$ kg/m^3 and $g = 9.8$ m/s^2, set up an integral for the work required to pump all the water out through the spout.





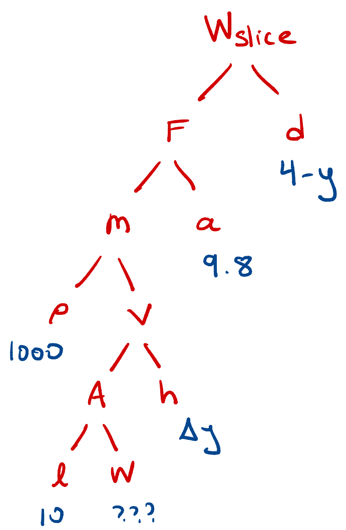
$$W_{slice} = 1000 \cdot 9.8 \cdot \pi \left(36 - (6-y)^2\right) \cdot (14-y)\, \Delta y$$

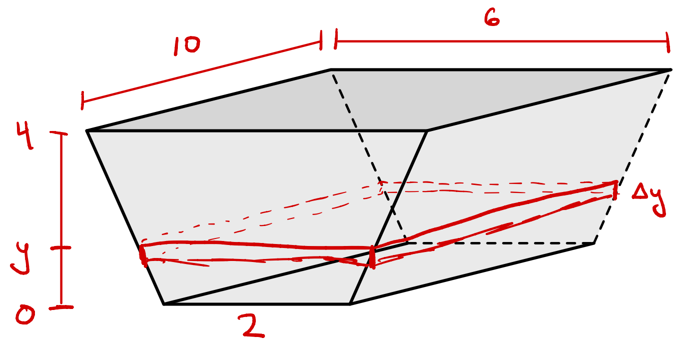
$$W_{Total} = \int_0^8 1000 \cdot 9.8\pi \left(36 - (6-y)^2\right) \cdot (14-y)\, dy$$

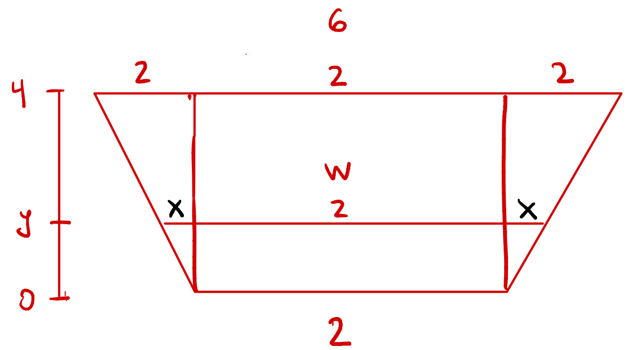
$$r_{slice} = \sqrt{36 - (6-y)^2}$$

(used $|6-y|$ since slice can be both above or below 6)

Problem. A trough is 10 m long. Its vertical cross-sections are trapezoids with height 4 m, bottom width 2 m, and top width 6 m. The trough is completely full of water. Using $\rho = 1000\ \text{kg/m}^3$ and $g = 9.8\ \text{m/s}^2$, set up an integral for the work required to pump all the water over the top edge.







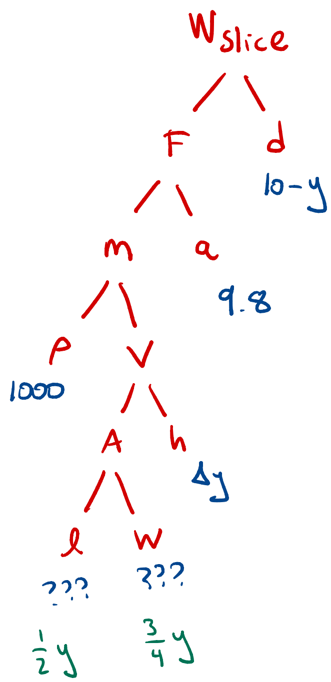
$$W_{slice} = 1000 \cdot 9.8 \cdot 10 \cdot (2+y) \cdot (4-y)\, \Delta y$$

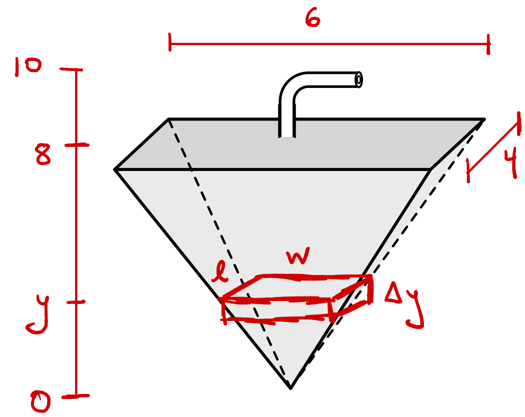
$$W_{Total} = \int_0^4 \underbrace{1000 \cdot 9.8}_{\rho \cdot g} \cdot \underbrace{10(2+y)}_{A(y)} \underbrace{(4-y)}_{d(y)}\, dy$$

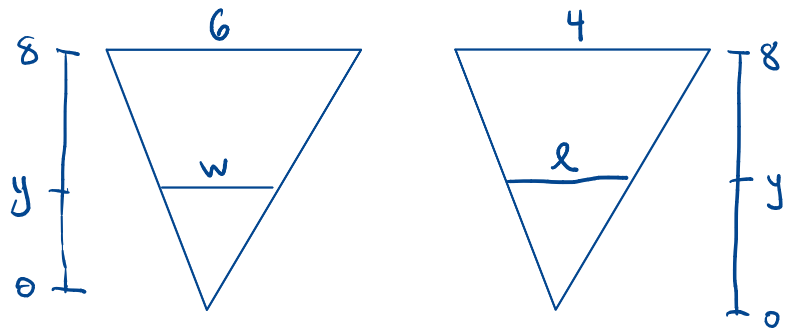
$$\frac{2}{4} = \frac{x}{y} \Rightarrow x = \frac{1}{2} y$$

$$w = \frac{1}{2}y + 2 + \frac{1}{2}y = 2 + y$$

Problem. A tank has the shape of an inverted rectangular pyramid with height 8 m. The top of the tank is a rectangle measuring 4 m by 6 m. The tank is completely full of water. A spout extends 2 m above the top of the tank. Using $\rho = 1000$ kg/m^3 and $g = 9.8$ m/s^2, set up an integral for the work required to pump all the water out through the spout.







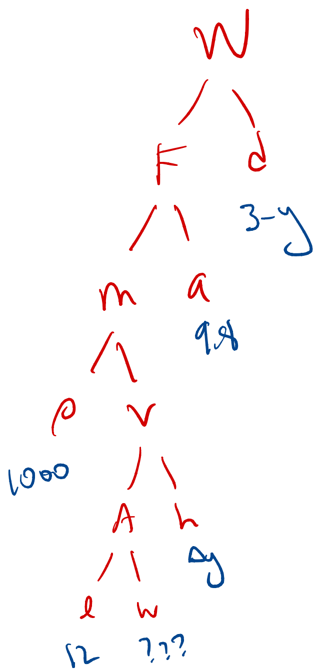
$$\frac{6}{8} = \frac{w}{y} \qquad \frac{4}{8} = \frac{l}{y}$$

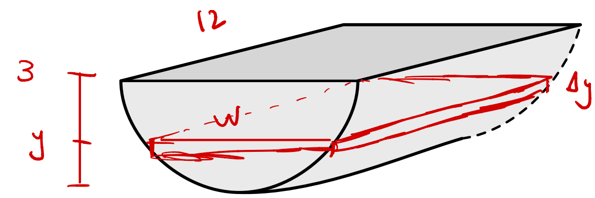
$$\frac{3}{4}y = w \qquad \frac{1}{2}y = l$$

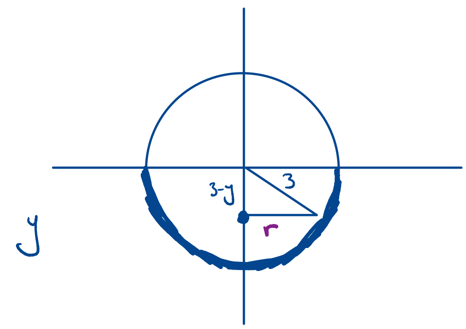
$$W_{slice} = 1000 \cdot 9.8 \cdot \frac{3}{8}y^2 \cdot (10-y)\,\Delta y$$

$$W_{Total} = \int_0^8 1000 \cdot 9.8 \cdot \frac{3}{8}y^2 (10-y)\,dy$$

Problem. A semi-cylindrical trough is 12 m long and has radius 3 m. The trough is completely full of water. Using $\rho = 1000$ kg/m^3 and $g = 9.8$ m/s^2, set up an integral for the work required to pump all the water over the top edge.







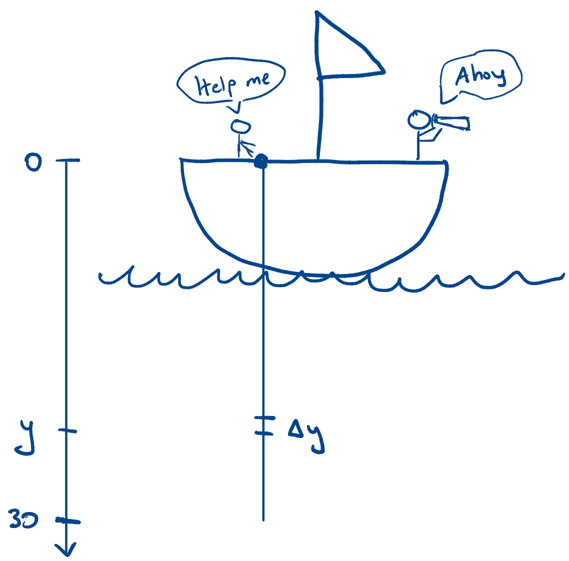
$w = 2 \cdot r_{slice}$

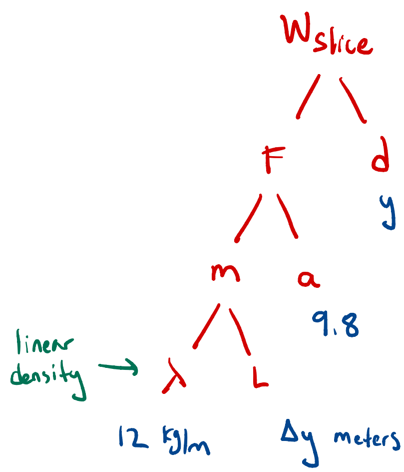
$w = 2 \cdot \sqrt{9-(3-y)^2}$

$$W_{slice} = 1000 \cdot 9.8 \cdot 12 \cdot 2\sqrt{9-(3-y)^2}\,(3-y)\,\Delta y$$

$$W_{Total} = \int_0^3 1000 \cdot 9.8 \cdot 12 \cdot 2\sqrt{9-(3-y)^2}\,(3-y)\,dy$$

Problem. A 30-meter-long anchor chain with a linear density of 12 kg/m is hanging from the side of a ship, with one end attached to the ship and the other submerged in the water. The chain is hoisted onto the deck of the ship. Compute the work required to lift the entire chain onto the ship.

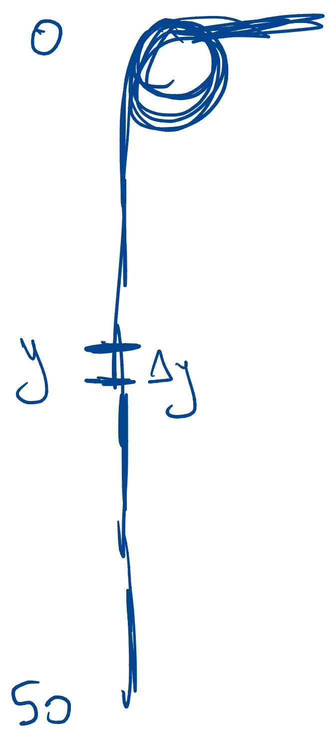


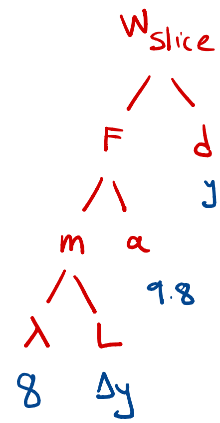


$$W_{slice} = 9.8 \cdot 12y \, \Delta y$$

$$W_{Total} = \int_0^{30} 9.8 \cdot 12y \, dy$$

Problem. A 50-meter-long chain with a linear density of 8 kg/m is hanging from a pulley at the top of a mine shaft. The chain is initially fully extended into the shaft and is lifted to the top. Compute the work required to lift the entire chain.

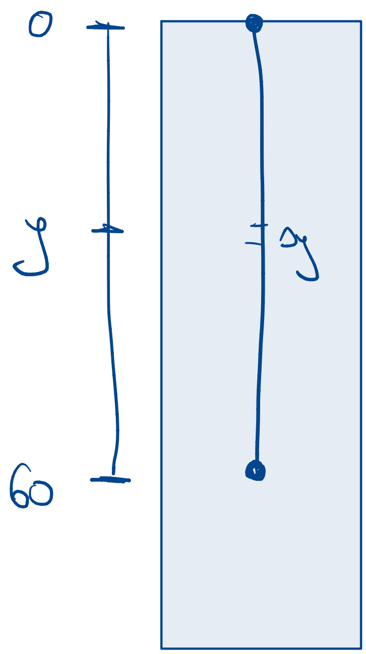


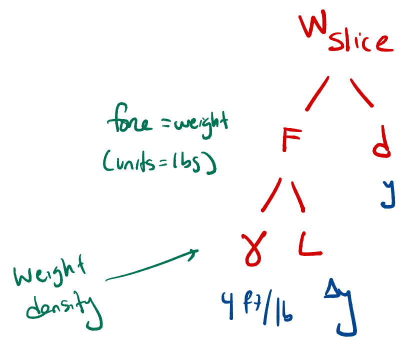


$$W_{slice} = 9.8 \cdot 8 y \, \Delta y$$

$$W_{Total} = \int_0^{50} 9.8 \cdot 8 y \, dy$$

Problem. A 60-ft cable weighing 240 lbs hangs vertically from the top of a building. The cable is pulled up to the top until the entire cable is on the roof. Compute the work required to lift the cable.



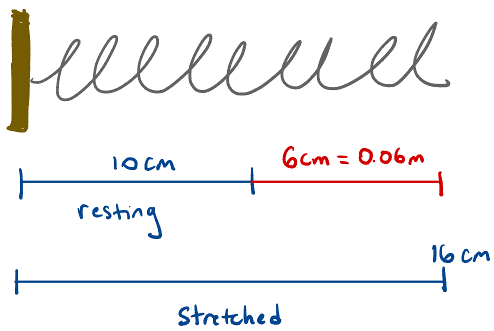


$$W_{slice} = 4\, y\, \Delta y$$

$$W_{Total} = \int_0^{60} 4\, y\, dy$$

is required to hold the spring at 16cm

Problem. A spring has natural length 10 cm. A force of 30 N stretches the spring to a length of 16 cm. How much work is required to stretch the spring from 12 cm to 18 cm?



① Find k:

$$F = kx$$

$$30 = k \cdot (0.06)$$

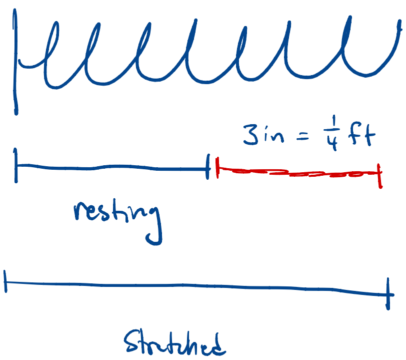
$$k = \frac{30}{0.06} = 500$$

② $W = \int_a^b kx \, dx$

$$W = \int_{0.02}^{0.08} 500x \, dx = \left[250x^2\right]_{0.02}^{0.08}$$

$$= 1.5 \text{ J}$$

Problem. A force of 12 lb is required to stretch a spring 3 in beyond its natural length. How much work is required to stretch the spring 8 in beyond its natural length?



1 Find k

$$F = kx$$

$$12 = k \cdot \frac{1}{4}$$

$$k = 48$$

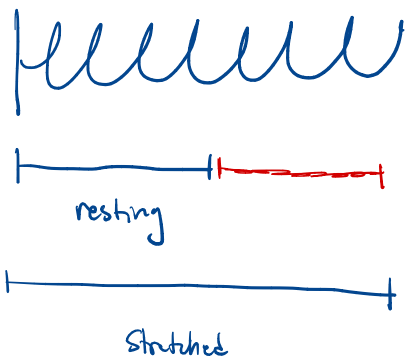
8in = $\frac{2}{3}$ ft

2 $$W = \int_0^{2/3} 48x \, dx$$

$$= \left[24x^2 \right]_0^{2/3}$$

$$= \frac{32}{3} \text{ ft-lb}$$

Problem. A spring requires 20 J of work to stretch it 0.4 m beyond its natural length. How much work is required to stretch it 0.7 m beyond its natural length?



① $$20 = \int_0^{0.4} kx \, dx$$

$$20 = \left[\frac{1}{2}kx^2\right]_0^{0.4}$$

$$20 = \frac{1}{2} \cdot k \cdot (0.4)^2$$

$$20 = 0.08\,k$$

$$k = 250$$

② $$W = \int_0^{0.7} 250x \, dx$$

$$= \left[125x^2\right]_0^{0.7}$$

$$= 125 \cdot (0.7)^2 = 61.25 \text{ J}$$