6.3 Volumes by Cylindrical Shells

The **cylindrical shell method** uses slices *parallel to the axis of rotation.* A thin slice revolves into a *cylindrical shell*, and we add up the shells to get the total volume.

$$V = 2\pi \int (\text{radius})(\text{height})(\text{thickness}).$$

If the axis of rotation is vertical, use vertical slices:

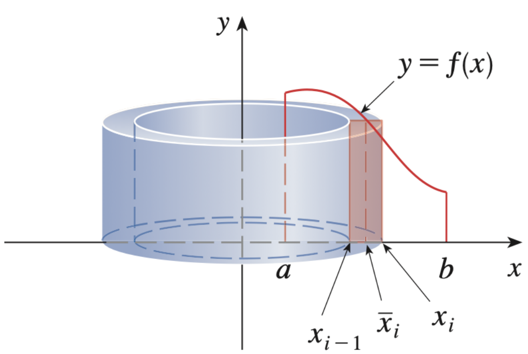
- *Thickness:* dx
- *Radius:* distance from x to the axis $x = a$: $r = |x - a|$
- *Height:* vertical length of the region at that x

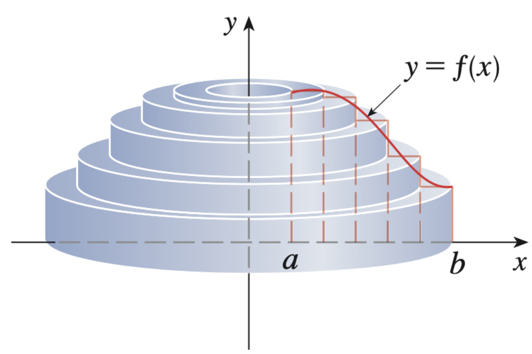
$$V = 2\pi \int_{x=a}^{x=b} r(x)\, h(x)\, dx.$$

If the axis of rotation is horizontal, use horizontal slices:

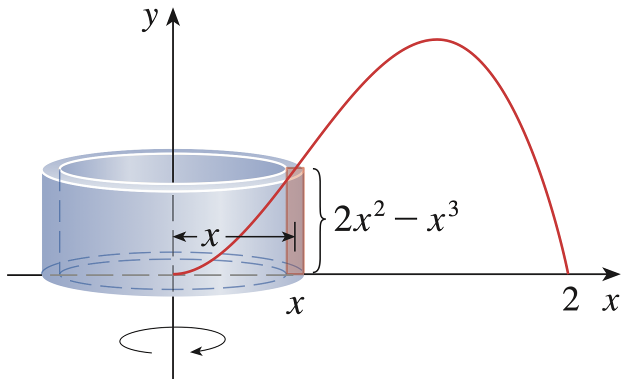
- *Thickness:* dy
- *Radius:* distance from y to the axis $y = b$: $r = |y - b|$
- *Height:* horizontal length of the region at that y

$$V = 2\pi \int_{y=c}^{y=d} r(y)\, h(y)\, dy.$$

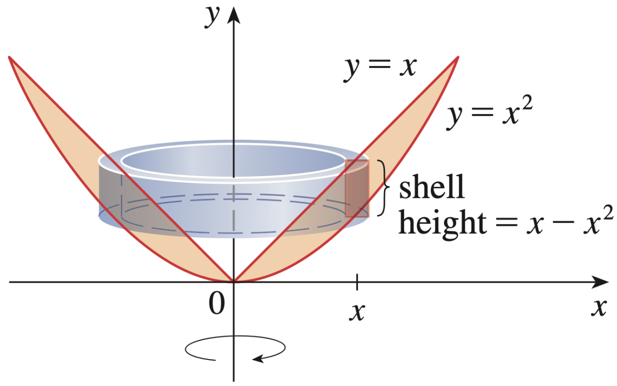




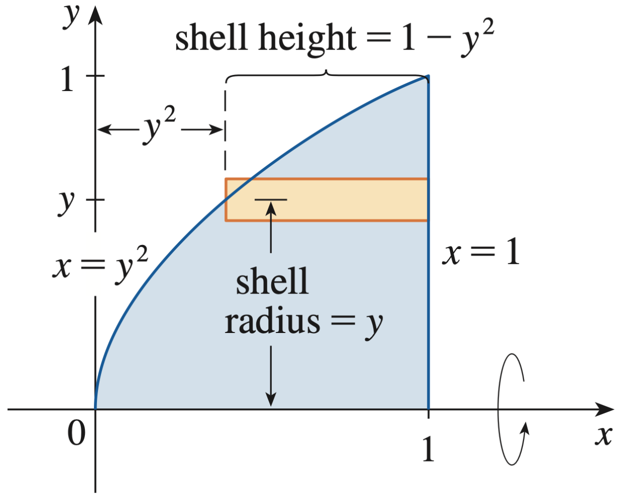
Example. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.



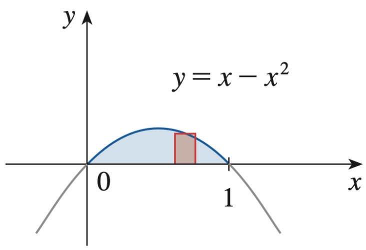
Example. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.

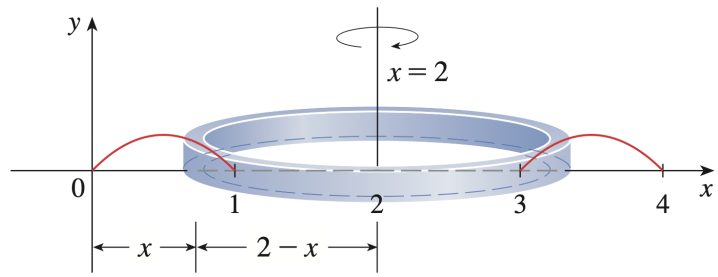


Example. Use cylindrical shells to find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.

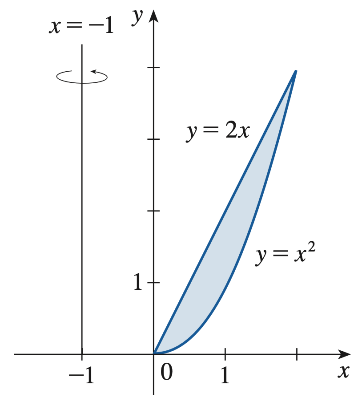


Example. Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.





Example. Consider the region in the first quadrant bounded by the curves $y = x^2$ and $y = 2x$. A solid is formed by rotating the region about the line $x = -1$.



Find the volume of the solid using:

(a) x as the variable of integration.

(b) y as the variable of integration.

Solution: