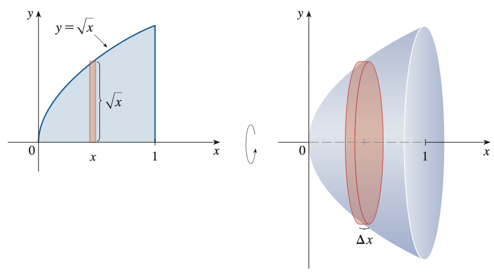
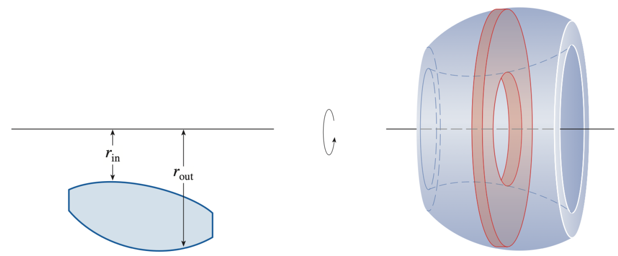
Mixed Practice: Disks, Washers, and Shells

There are three primary slicing methods used to compute the volume of a solid of revolution:

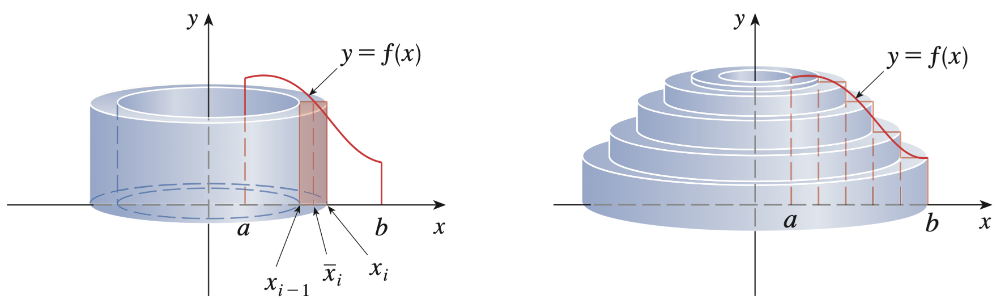
Disk Method: Slices are drawn perpendicular to the axis of rotation. If there is no gap between the region and the axis, each slice is disk.



Washer Method: Slices are drawn perpendicular to the axis of rotation. If there is a gap between the region and the axis, each slice is washer.



Shell Method: Slices are drawn parallel to the axis of rotation. Each slice forms a cylindrical shell, and its volume is determined by its radius, height, and thickness.



This flowchart helps you choose the correct setup for a volume of revolution problem.

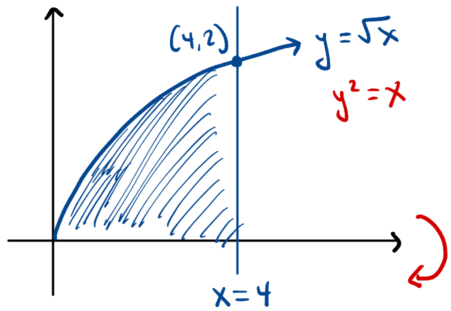
Draw a line between boundary curves

- Perpendicular to axis of rotation → Gap to Axis?
  - No → **Disk Method**: $$\pi \int_a^b [R(x)]^2 \, dx$$
  - Yes → **Washer Method**: $$\pi \int_a^b \left(R(x)^2 - r(x)^2\right) dx$$
- Parallel to axis of rotation → **Shell Method**: $$\int_a^b 2\pi r(x) \cdot h(x) \, dx$$

Problem 1. Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x}, \qquad y = 0, \qquad x = 4$$

about the line $y = 0$.



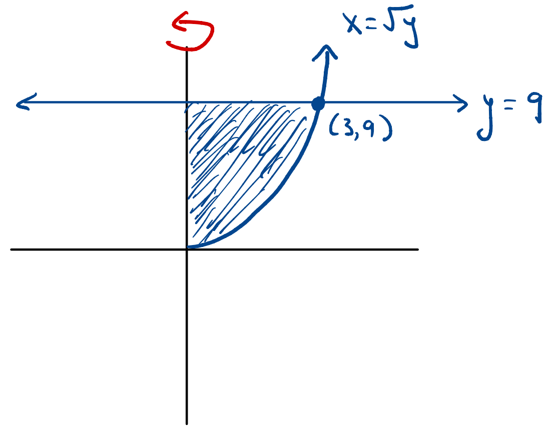
Disks: $\int_0^4 \pi (\sqrt{x})^2 \, dx$

Shells: $\int_0^2 2\pi y \cdot (4 - y^2) \cdot dy$

Problem 2. Find the volume of the solid obtained by rotating the region bounded by

$$x = \sqrt{y}, \qquad x = 0, \qquad y = 9$$

about the line $x = 0$.



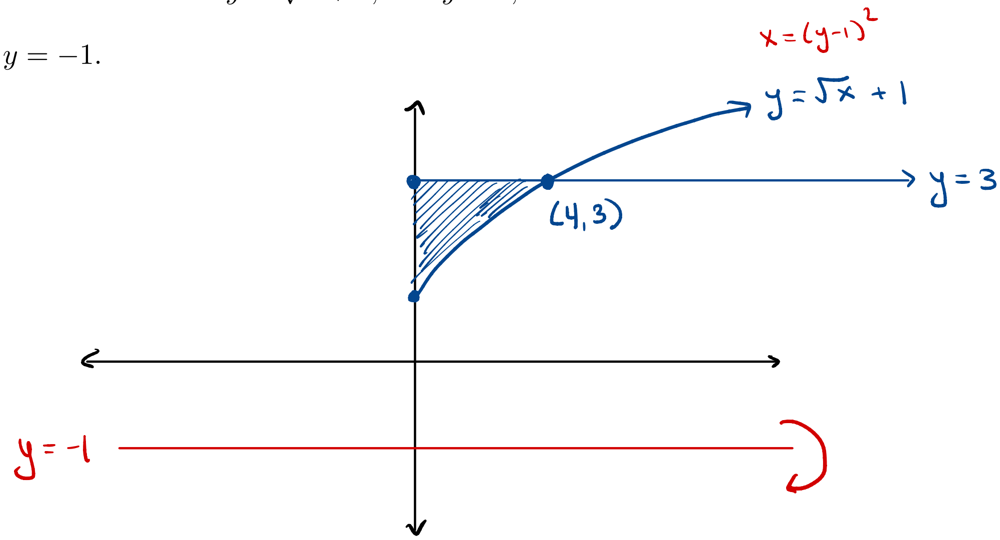
Disks: $\int_0^9 \pi (\sqrt{y})^2 \, dy$

Shells: $\int_0^3 2\pi x \cdot (9 - x^2) \, dx$

Problem 3. Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x} + 1, \qquad y = 3, \qquad x = 0$$

about the line $y = -1$.



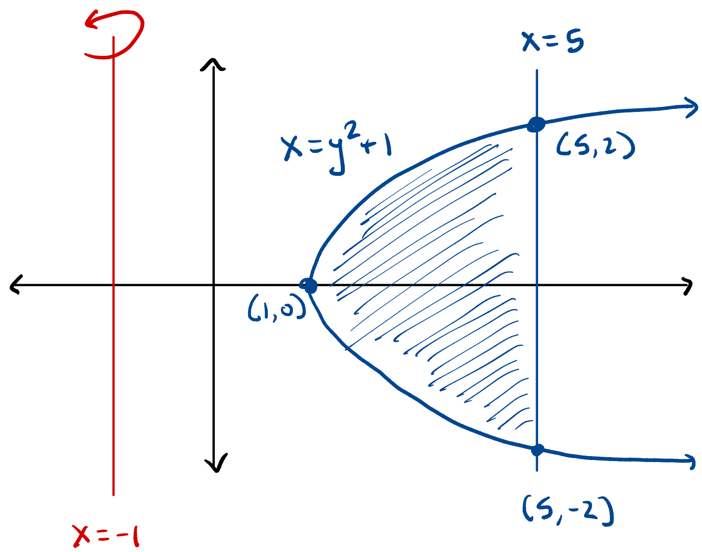
Washers: $\int_0^4 \pi (3+1)^2 - \pi (\sqrt{x}+1+1)^2 \, dx$

Shells: $\int_1^3 2\pi (y+1) \cdot (y-1)^2 \, dy$

Problem 4. Find the volume of the solid obtained by rotating the region bounded by

$$x = y^2 + 1 \qquad \text{and} \qquad x = 5$$

about the line $x = -1$.



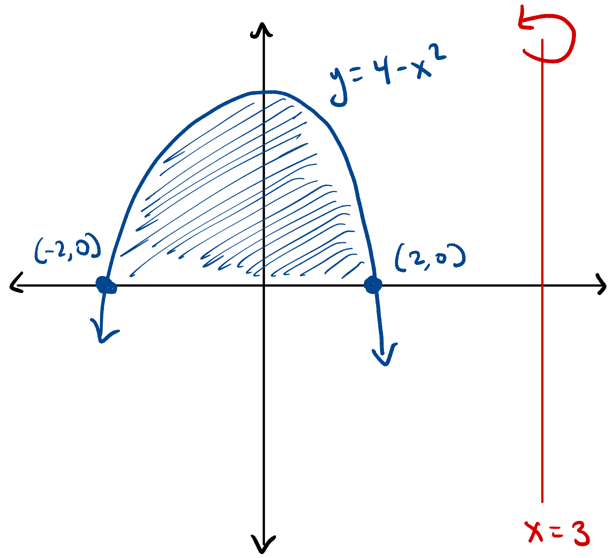
Washers: $\int_{-2}^{2} \pi(5+1)^2 - \pi(y^2+1+1)^2 \, dy$

Shells: X

Problem 5. Find the volume of the solid obtained by rotating the region bounded by

$$y = 4 - x^2 \qquad \text{and} \qquad y = 0$$

about the line $x = 3$.



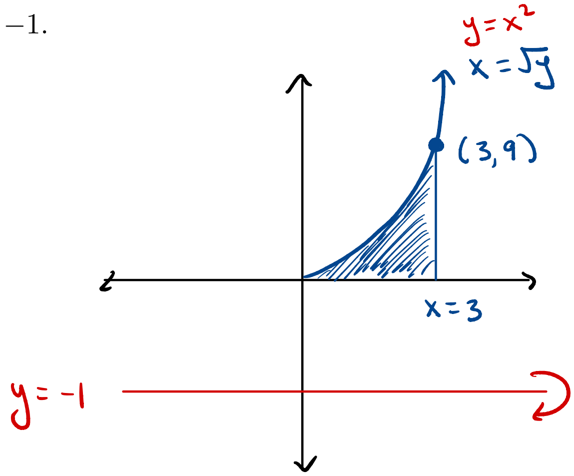
Washers: ✗

Shells: $$\int_{-2}^{2} \underbrace{2\pi(3-x)}_{\text{circumference}} \cdot \underbrace{(4-x^2)}_{\text{height}} \, \underbrace{dx}_{\text{thickness}}$$

Problem 6. Find the volume of the solid obtained by rotating the region bounded by

$$x = \sqrt{y}, \qquad x = 3, \qquad y = 0$$

about the line $y = -1$.



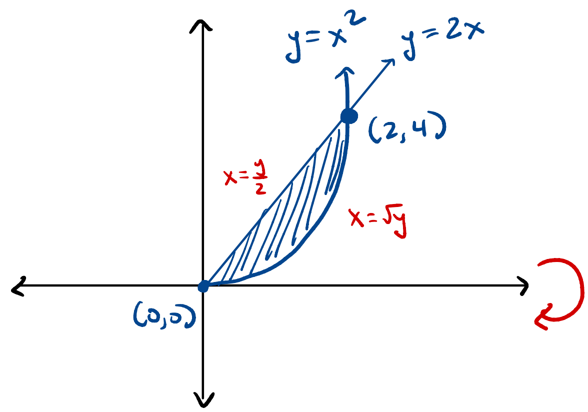
Washers: $\int_0^3 \pi (x^2+1)^2 - \pi (1)^2 \, dx$

Shells: $\int_0^9 2\pi (y+1) \cdot (3-\sqrt{y}) \, dy$

Problem 7. Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 \qquad \text{and} \qquad y = 2x$$

about the line $y = 0$.



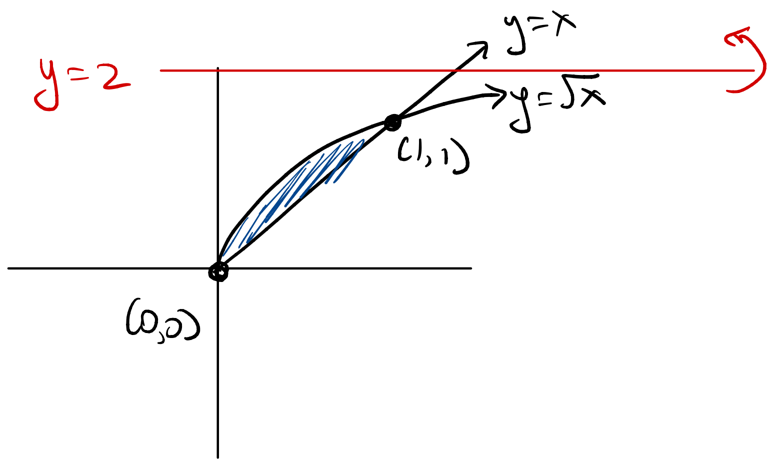
Washers: $\int_0^2 \pi(2x)^2 - \pi(x^2)^2\,dx$

Shells: $\int_0^4 2\pi y \cdot \left(\sqrt{y} - \frac{y}{2}\right) dy$

Problem 8. Find the volume of the solid obtained by rotating the region bounded by

$$y = x \qquad \text{and} \qquad y = \sqrt{x}$$

about the line $y = 2$.



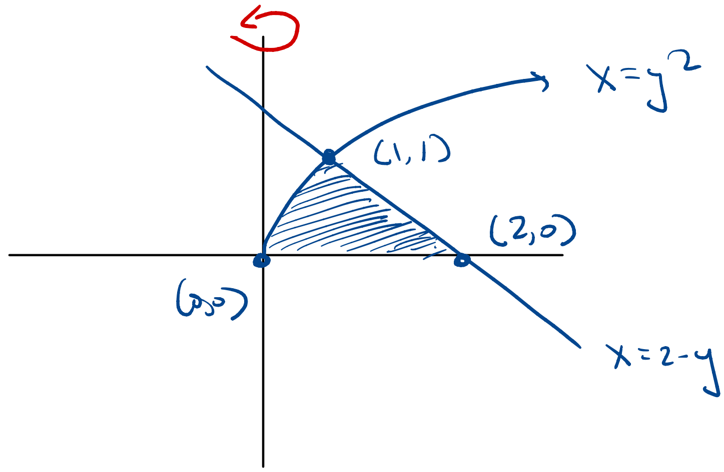
Washers: $\int_0^1 \pi(2-x)^2 - \pi(2-\sqrt{x})^2\, dx$

Shells: $\int_0^1 2\pi(2-y)\cdot(y-y^2)\, dy$

Problem 9. Find the volume of the solid obtained by rotating the region bounded by

$$x = y^2, \qquad x = 2 - y, \qquad y = 0$$

about the line $x = 0$.



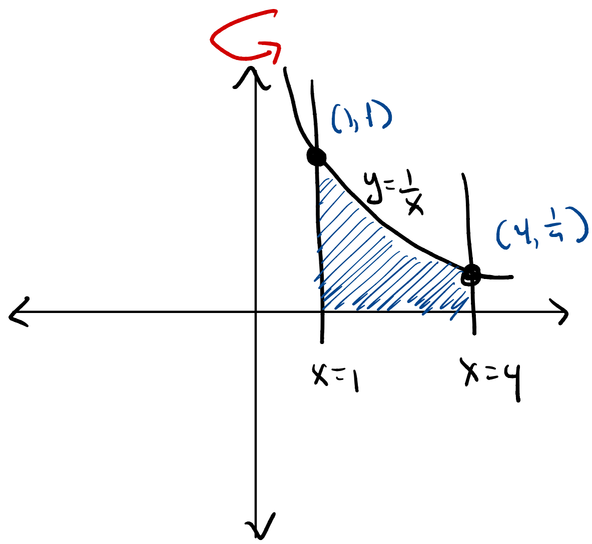
Washers: $\int_0^1 \pi(2-y)^2 - \pi(y^2)^2 \, dy$

Shells: X

Problem 10. Find the volume of the solid obtained by rotating the region bounded by

$$y = \frac{1}{x}, \qquad y = 0, \qquad x = 1, \qquad x = 4$$

about the line $x = 0$.



Washers: X

Shells: $\int_1^4 2\pi x \cdot \frac{1}{x} \cdot dx$