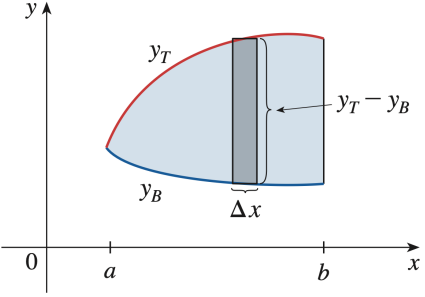
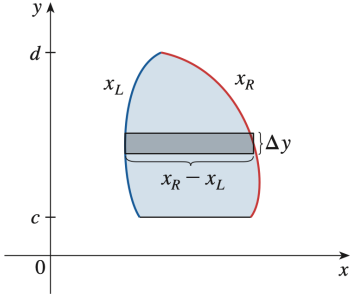
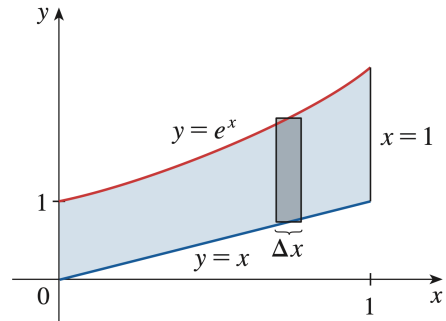


## 6.1 Areas Between Curves

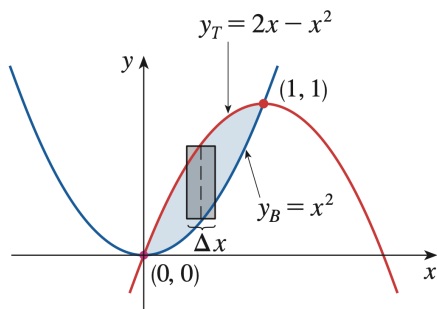
Integration with Respect to $x$	Integration with Respect to $y$
<p data-bbox="233 338 797 443">Choose to integrate with respect to <math>x</math> if the region is vertically bounded (i.e., top and bottom curves).</p>  <p data-bbox="233 804 505 835">The area is given by:</p> $\text{Area} = \int_a^b (y_T - y_B) dx$	<p data-bbox="824 338 1388 443">Choose to integrate with respect to <math>y</math> if the region is horizontally bounded (i.e., right and left curves).</p>  <p data-bbox="824 804 1096 835">The area is given by:</p> $\text{Area} = \int_c^d (x_R - x_L) dy$

### Area Between Curves: Integrating With Respect to $x$

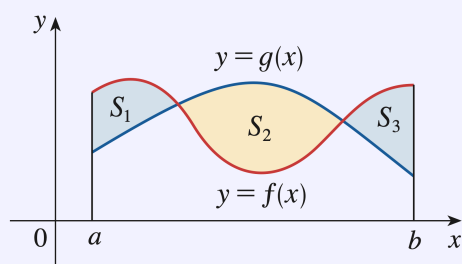
**Example.** Find the area of the region bounded above by  $y = e^x$ , bounded below by  $y = x$ , and bounded on the sides by  $x = 0$  and  $x = 1$ .



**Example.** Find the area of the region enclosed by the parabolas  $y = x^2$  and  $y = 2x - x^2$ .



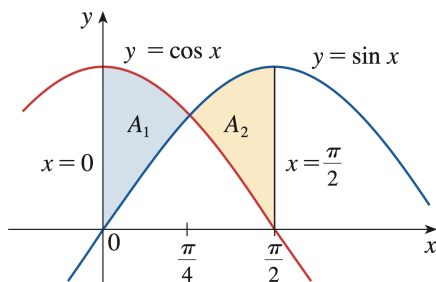
Sometimes we are asked to find the area between the curves  $y = f(x)$  and  $y = g(x)$  where  $f(x) \geq g(x)$  for some values of  $x$  but  $g(x) \geq f(x)$  for other values of  $x$ .



In this case, we split the given region  $S$  into several regions  $S_1, S_2, \dots$  with areas  $A_1, A_2, \dots$ . The total area is

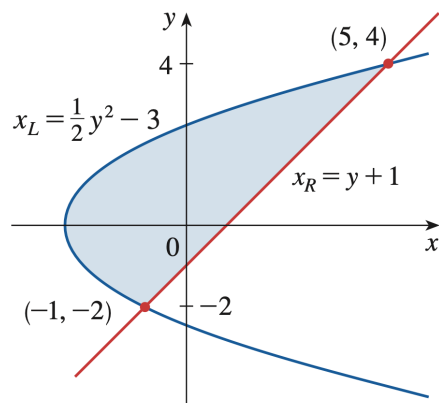
$$A = A_1 + A_2 + \dots = \int_a^b |f(x) - g(x)| dx$$

**Example.** Find the area of the region bounded by the curves  $y = \sin x$ ,  $y = \cos x$ ,  $x = 0$ , and  $x = \pi/2$ .



### Area Between Curves: Integrating With Respect to $y$

**Example.** Find the area enclosed by the line  $y = x - 1$  and the parabola  $y^2 = 2x + 6$ .



**Example.** Find the area of the region enclosed by the curves  $y = 1/x$ ,  $y = x$ , and  $y = \frac{1}{4}x$ , using

(a)  $x$  as the variable of integration.

(b)  $y$  as the variable of integration.

