Quiz 9 Outline

Format. This quiz has **3 multiple choice** question and **1 free-response** questions.

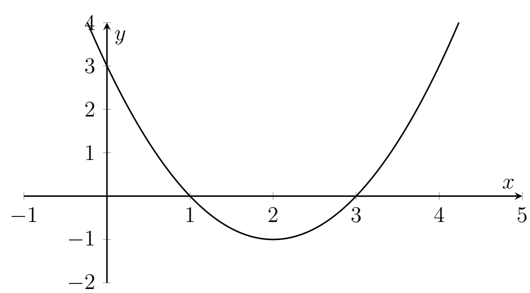
1. Compute the absolute maximum / minimum of a function using the closed interval method.

> **Example:** Where does the absolute minimum of the function $f(x) = \ln\left(x^2 + 1\right)$ occur on the interval $[-1, 1]$?
>
> (a) $x = -1$
>
> (b) $x = 0$
>
> (c) $x = 1$
>
> (d) $x = 2$
>
> (e) Does not exist

2. Conceptual question about Rolle's Theorem or the Mean Value Theorem.

> **Example:** Suppose f is a function which is continuous and differentiable on the interval $[-1, 1]$. Which of the following conditions would guarantee that $f'(c) = 0$ for some c between -1 and 1?
>
> (a) f is a polynomial.
>
> (b) $f(-1) = f(1)$.
>
> (c) f has no horizontal tangent lines.
>
> (d) f has a change in concavity.
>
> (e) f is increasing.

3. Apply the Mean Value Theorem.

> **Example:** The following is a graph of f.
>
> 
>
>
> For what value of x does f satisfy the Mean Value Theorem on the interval $[1, 3]$?
>
> (a) 3
>
> (b) -1
>
> (c) 2
>
> (d) 1
>
> (e) 0

4. Linear Approximation.

Example: Let $f(x) = \ln(x)$.

(a) Use a linearization to estimate $\ln(0.99)$.

(b) Is your estimate from part (a) an overestimate or an underestimate? Provide a justification.

Solutions

Solution: $\boxed{\text{(b)}}$

$$f'(x) = \frac{2x}{x^2+1} = 0 \iff x = 0.$$

Evaluate f at the endpoints and the critical point:

$$f(-1) = \ln 2, \quad f(0) = \ln 1 = 0, \quad f(1) = \ln 2.$$

The *absolute minimum* on $[-1, 1]$ is 0 at $x = \boxed{0}$ and the absolute maximum is $\ln 2$ at $x = \pm 1$.

Solution: $\boxed{\text{(b)}}$

By Rolle's Theorem, if f is continuous on $[-1, 1]$, differentiable on $(-1, 1)$, and $f(-1) = f(1)$, then there exists c in $(-1, 1)$ with $f'(c) = 0$.

Solution: $\boxed{\text{(c)}}$

On $[1, 3]$, the secant slope is

$$\frac{f(3) - f(1)}{3 - 1} = \frac{0 - 0}{2} = 0.$$

By the Mean Value Theorem, there exists c in $(1, 3)$ with $f'(c) = 0$. The displayed parabola has a horizontal tangent at $c = 2$.

Solution:

(a) Linearize at $a = 1$:

$$\begin{aligned} L(x) &= f(1) + f'(1)(x - 1) \\ &= 0 + 1 \cdot (x - 1) \\ &= x - 1. \end{aligned}$$

Hence $\ln(0.99) = f(0.99) \approx L(0.99) = 0.99 - 1 = \boxed{-0.01}$.

(b) Since $f''(x) = -\dfrac{1}{x^2} < 0$ for $x > 0$, $\ln x$ is *concave down*. Therefore the tangent line lies *above* the curve near $x = 1$, so $L(0.99)$ is an *overestimate*.