Math 1300 **Name:** ______________________

Fall 2025

Quiz 7

1. (5 points) Find the equation of the tangent line to $y^2 + x^3 = xe^{3y}$ at $(1, 0)$.

Solution.

1. **Differentiate implicitly with respect to x.**

$$\frac{d}{dx}\big(y^2 + x^3\big) = \frac{d}{dx}\big(xe^{3y}\big) \quad \Longrightarrow \quad 2y\, y' + 3x^2 = e^{3y} + x\big(e^{3y} \cdot 3y'\big).$$

Rearranging to solve for y':

$$(2y)\, y' - 3xe^{3y}\, y' = e^{3y} - 3x^2 \quad \Longrightarrow \quad y' = \frac{e^{3y} - 3x^2}{2y - 3xe^{3y}}.$$

2. **Evaluate the slope at $(1, 0)$.**

$$y'(1, 0) = \frac{e^0 - 3 \cdot 1^2}{2 \cdot 0 - 3 \cdot 1 \cdot e^0} = \frac{1 - 3}{0 - 3} = \frac{-2}{-3} = \frac{2}{3}.$$

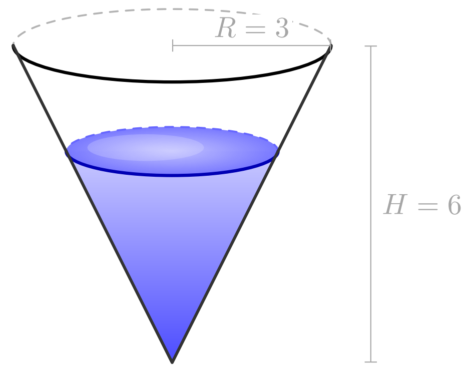
So the tangent slope is $m = \frac{2}{3}$.

3. **Write the tangent line through $(1, 0)$ with slope $m = \frac{2}{3}$.**

$$y - 0 = \frac{2}{3}\,(x - 1) \quad \Longrightarrow \quad \boxed{y = \frac{2}{3}\,(x - 1)}.$$

2. (5 points) A right circular conical tank has height 6 m and top radius 3 m. Water is being pumped in so that the water depth h is rising at a constant rate of 0.25 m/min. How fast is the volume of water in the tank increasing at the instant when the water depth is $h = 4$ m?

Volume formula for a cone: $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$.



Solution.

1. Given: $\frac{dh}{dt} = 0.25$. Want: $\frac{dV}{dt}$ when $h = 4$ m.
2. Relate V and h:

$$V = \frac{1}{3}\pi r^2 h$$

To get V in terms of h only, use similar triangles:

$$\frac{r}{h} = \frac{R}{H} = \frac{3}{6} = \frac{1}{2} \quad \Longrightarrow \quad r = \frac{h}{2}.$$

Therefore,

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h = \frac{\pi}{12} h^3.$$

3. Differentiating both sides, we obtain:

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{\pi}{12}h^3\right) = \frac{\pi}{4} h^2 \frac{dh}{dt}.$$

4. At $h = 4$ m **and** $\frac{dh}{dt} = 0.25$ m/min,

$$\frac{dV}{dt} = \frac{\pi}{4}(4)^2(0.25) = \frac{\pi}{4} \cdot 16 \cdot \frac{1}{4} = \boxed{\pi \text{ m}^3/\text{min}}.$$