Math 1300
Fall 2025
Quiz 6

Name: Champ

1. (2 points) If $f(x) = \cos(\tan(x))$, compute $f'(x)$.

   (a) $f'(x) = \cos^2(x) - \sin^2(x)$

   (b) $f'(x) = \cos(x)\cos(\tan(x))$

   (c) $f'(x) = -\sin(x)\cos(\tan(x))$

   (d) $f'(x) = -\sin(\tan(x))\sec^2(x)$

   (e) $f'(x) = -\sin(\sec^2(x))$

$y = \cos(u) \qquad u = \tan(x)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= -\sin(u) \cdot \sec^2(x)$$

$$= -\sin(\tan(x)) \cdot \sec^2(x)$$

2. (3 points) Compute $\frac{d}{dx}\left(\sin(e^{\sqrt{x}})\right)$.

$y = \sin(u) \qquad u = e^v \qquad v = \sqrt{x}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

$$= \cos(u) \cdot e^v \cdot \frac{1}{2}x^{-1/2}$$

$$= \cos(e^v) \cdot e^v \cdot \frac{1}{2}x^{-1/2}$$

$$= \cos(e^{\sqrt{x}}) \cdot e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}$$

Answer: $\boxed{\cos(e^{\sqrt{x}}) \cdot e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}}$

3. (5 points) Consider the curve defined by $y^2 = \sin(x+y)$.

(a) (3 points) Use implicit differentiation to compute $\dfrac{dy}{dx}$.

$$\frac{d}{dx}(y^2) = \frac{d}{dx}\left(\sin(x+y)\right)$$

↓ chain rule ↓ chain rule

$$2y \cdot \frac{d}{dx}(y) = \cos(x+y) \cdot \frac{d}{dx}(x+y)$$

$$2y \cdot \frac{dy}{dx} = \cos(x+y) \cdot \left(1 + \frac{dy}{dx}\right)$$

$$2y \cdot \frac{dy}{dx} = \cos(x+y) + \cos(x+y)\frac{dy}{dx}$$

$$2y\frac{dy}{dx} - \cos(x+y)\frac{dy}{dx} = \cos(x+y)$$

$$\frac{dy}{dx}\left(2y - \cos(x+y)\right) = \cos(x+y)$$

$$\frac{dy}{dx} = \frac{\cos(x+y)}{2y - \cos(x+y)}$$

(b) (2 points) Find the equation of the tangent line to the curve at the point $(\pi, 0)$.

At $(\pi, 0)$, $\dfrac{dy}{dx} = \dfrac{\cos(\pi+0)}{2\cdot 0 - \cos(\pi+0)} = \dfrac{-1}{0-(-1)} = -1$

$$\boxed{y - 0 = -1(x - \pi)}$$