Math 1300
Fall 2025
Quiz 5

Name: Champ

1. (1 point) Let $f(x) = x^3 e^x$. Find **all** values of x for which $f(x)$ has a horizontal tangent.

   (a) $x = -3,\ 0$
   (b) $x = -3$
   (c) $x = 0$
   (d) $x = -3,\ 0,\ 3$
   (e) none

$$f'(x) = x^3 \cdot \frac{d}{dx}(e^x) + e^x \cdot \frac{d}{dx}(x^3) = x^3 \cdot e^x + e^x \cdot 3x^2$$
$$= e^x \cdot (x^3 + 3x^2)$$
$$= e^x \cdot x^2 \cdot (x+3)$$

$$0 = e^x \cdot x^2 \cdot (x+3) \Rightarrow x = 0 \text{ or } x = -3$$

2. (1 point) Compute $\dfrac{d}{dx}\left(\dfrac{x^2+1}{e^x}\right)$.

   (a) $\dfrac{2x}{e^x}$
   (b) $\dfrac{2x + x^2 + 1}{e^x}$
   (c) $\dfrac{2x - x^2 - 1}{e^x}$
   (d) $-\dfrac{x^2+1}{e^x}$
   (e) $\dfrac{2x - x^2 - 1}{e^{2x}}$

$$\frac{e^x \cdot \frac{d}{dx}(x^2+1) - (x^2+1) \cdot \frac{d}{dx}(e^x)}{(e^x)^2}$$

$$= \frac{\cancel{e^x} \cdot 2x - (x^2+1) \cdot \cancel{e^x}}{(e^x)^{\cancel{2}}}$$

$$= \frac{2x - x^2 - 1}{e^x}$$

3. (2 points) Compute $f'(1)$, given that $f(x) = \dfrac{\sqrt[3]{x} - \sqrt{x}}{x}$.

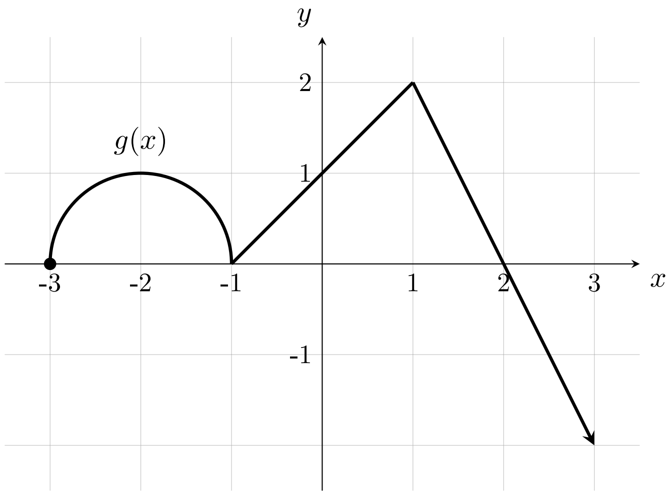
   (a) $-\dfrac{7}{6}$
   (b) $-\dfrac{1}{6}$
   (c) $-\dfrac{1}{3}$
   (d) $\dfrac{1}{6}$
   (e) 0

$$f(x) = \frac{x^{1/3} - x^{1/2}}{x} = x^{-2/3} - x^{-1/2}$$

$$f'(x) = -\frac{2}{3}x^{-5/3} + \frac{1}{2}x^{-3/2}$$

$$f'(1) = -\frac{2}{3} + \frac{1}{2} = -\frac{4}{6} + \frac{3}{6} = -\frac{1}{6}$$

4. (6 points) Use the following graph and table to answer the questions below.



x	$f(x)$	$f'(x)$
-2	1	-2
-1	-1	1
0	4	-3
1	1	2
2	-1	3

(a) If $P(x) = \dfrac{f(x)}{g(x)}$, find $P'(0)$.

(a) -7
(b) -3
(c) 1
(d) 4
(e) 7

$$\frac{g(0)f'(0) - f(0)g'(0)}{(g(0))^2}$$

$$\frac{1\cdot -3 - 4\cdot 1}{1^2} = -3-4 = -7$$

(b) If $Q(x) = xf(x)$, find $Q'(-2)$.

(a) -3
(b) -1
(c) 2
(d) 4
(e) 5

$Q'(x) = x\cdot f'(x) + f(x)\cdot 1$

$Q'(-2) = -2\cdot -2 + 1\cdot 1$

$= 4+1$

(c) If $R(x) = \dfrac{f(x)g(x)}{x}$, find $R'(2)$.

(a) -1
(b) 0
(c) $\frac{1}{2}$
(d) 1
(e) 2

$$\frac{x\cdot[f(x)g'(x)+g(x)f'(x)] - f(x)g(x)}{x^2}$$

$$\frac{2\cdot[-1\cdot -2 + 0] - (-1)\cdot 0}{4}$$

$\frac{4}{4} = 1$