1. (6 points) Consider the function

$$f(x) = \frac{(3x^2 + 4x + 1)(x - 2)}{(x + 1)(x^2 - 4)}.$$

(a) (2 points) Find the horizontal asymptote(s), if they exist.

Expanded:
$$\frac{3x^3+4x^2+x-6x^2-8x-2}{x^3-4x+x^2-4} = \frac{3x^3-2x^2-7x-2}{x^3+x^2-4x-4}$$

Answer:
$$y = 3$$

(b) (2 points) Find the vertical asymptote(s), if they exist.

Factored:
$$\frac{(3x+1)(x+1)(x-2)}{(x+1)(x+2)(x+2)}$$

(c) (2 points) Determine whether f(x) has any removable discontinuities (holes). If so, express each as an ordered pair in the form (x, y).

Simplified:
$$y = \frac{3x+1}{x+2}$$

When
$$x=-1$$
, $y=\frac{-2}{1}$

when
$$x = 2$$
, $y = \frac{7}{4}$

2. (4 points) Use the limit definition of the derivative to find f'(10), where

$$f(x) = \sqrt{x - 1}.$$

$$f'(10) = \lim_{h \to 0} \frac{f(10+h) - f(10)}{h} = \lim_{h \to 0} \frac{\sqrt{10+h-1} - \sqrt{10-1}}{h}$$

=
$$\lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h} \cdot \frac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3}$$

=
$$\lim_{h\to 0} \frac{q_{+h}-q}{h(\sqrt{q_{+h}}+3)}$$

$$= \frac{1}{\sqrt{9+3}}$$