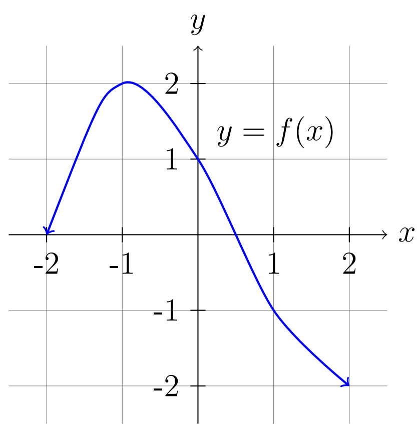
Quiz 14 Outline

Format. This quiz has **2 short-answer** questions, **1 multiple-choice** question, and **1 free response** question.

Use the following graph of $f(x)$ to answer the questions below.



1. Use the Evaluation Theorem to evaluate a definite integral from a graph.

Example: Evaluate $\int_{-2}^{0} f'(x)\,dx$.

Answer:

2. Use the Fundamental Theorem of Calculus (Part 1) to differentiate an accumulation function.

Example: Let

$$G(x) = \int_{-2}^{x} f(t)\,dt.$$

Use the graph to evaluate $G'(1)$.

Answer:

3. Find the area of a region bounded by two curves.

Example: Find the area of the region bounded by

$$f(x) = x^2 \quad \text{and} \quad g(x) = 3x.$$

(A.) $\frac{3}{2}$

(B.) $\frac{9}{4}$

(C.) $\frac{9}{2}$

(D.) 6

(E.) 9

4. Evaluate a definite integral using a u-substitution.

Example: Evaluate the integral

$$\int_0^1 \frac{x^2}{\sqrt{x^3+4}}\,dx.$$

Solution: By the Evaluation Theorem (FTC, Part 2),

$$\int_{-2}^{0} f'(x)\,dx = f(0) - f(-2).$$

From the graph, we read off

$$f(-2) = 0 \quad \text{and} \quad f(0) = 1.$$

Therefore,

$$\int_{-2}^{0} f'(x)\,dx = 1 - 0 = 1.$$

Solution: We are given

$$G(x) = \int_{-2}^{x} f(t)\,dt.$$

By the Fundamental Theorem of Calculus (Part 1),

$$G'(x) = f(x).$$

Therefore,

$$G'(1) = f(1).$$

From the graph, $f(1) = -1$, so

$$G'(1) = -1.$$

Solution: We are asked for the area of the region bounded by

$$f(x) = x^2 \quad \text{and} \quad g(x) = 3x.$$

First find the intersection points:

$$x^2 = 3x \quad \Longrightarrow \quad x^2 - 3x = 0 \quad \Longrightarrow \quad x(x-3) = 0,$$

so $x = 0$ and $x = 3$. On the interval $[0, 3]$, the line $g(x) = 3x$ lies above the parabola $f(x) = x^2$. Thus the area is

$$\begin{aligned} A = \int_0^3 \big[g(x) - f(x)\big]\,dx = \int_0^3 (3x - x^2)\,dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 &= \left(\frac{3 \cdot 3^2}{2} - \frac{3^3}{3}\right) - (0 - 0) \\ &= \left(\frac{27}{2} - 9\right) \\ &= \frac{27}{2} - \frac{18}{2} \\ &= \frac{9}{2}. \end{aligned}$$

So the area is $\frac{9}{2}$, which corresponds to choice (C).

Solution: We need to evaluate

$$\int_0^1 \frac{x^2}{\sqrt{x^3+4}}\,dx.$$

Use the substitution

$$u = x^3 + 4, \quad \frac{du}{dx} = 3x^2 \quad \Longrightarrow \quad du = 3x^2\,dx,$$

so

$$x^2\,dx = \frac{1}{3}\,du.$$

Change the limits:

$$x = 0 \implies u = 0^3 + 4 = 4, \qquad x = 1 \implies u = 1^3 + 4 = 5.$$

The integral becomes

$$\begin{aligned}
\int_0^1 \frac{x^2}{\sqrt{x^3+4}}\,dx &= \int_4^5 \frac{1}{\sqrt{u}} \cdot \frac{1}{3}\,du \\
&= \frac{1}{3}\int_4^5 u^{-1/2}\,du \\
&= \frac{1}{3}\left[2u^{1/2}\right]_4^5 \\
&= \frac{2}{3}\left(\sqrt{5} - \sqrt{4}\right) \\
&= \frac{2}{3}(\sqrt{5} - 2).
\end{aligned}$$