Math 1300
Fall 2025
Quiz 13

Name: Champ

1. (2 points) Use the information in the table to evaluate $\int_1^4 f'(z)\,dz$.

z	0	1	2	3	4	5
$f(z)$	5	2	0	3	4	9
$f'(z)$	-1	0	3	1	-2	4
$f''(z)$	2	1	-1	-2	0	1

(a) -4
(b) -2
(c) 0
(d) 2
(e) 4

$$\int_1^4 f'(z)\,dz = f(4) - f(1) = 4 - 2 = 2$$

Not Graded

2. (2 points) Evaluate $\int \frac{6}{x^2+9}\,dx$.

Will see how to do this in §5.5 (u-substitution)

(a) $6\arctan\left(\frac{x}{3}\right) + C$
(b) $2\arctan\left(\frac{x}{3}\right) + C$
(c) $\frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$
(d) $2\arctan(3x) + C$
(e) $\arctan\left(\frac{x}{9}\right) + C$

$$\int \frac{6}{x^2+9}\,dx = \int \frac{6}{9\cdot\left(\frac{x^2}{9}+1\right)}\,dx$$

$$= \frac{2}{3}\cdot\int \frac{1}{\left(\frac{x}{3}\right)^2+1}\,dx$$

Recognize $\frac{1}{\left(\frac{x}{3}\right)^2+1}$ as the derivative of $\arctan\left(\frac{x}{3}\right)\cdot 3$ ← chain rule

3. (2 points) Let $r(t)$ be the snowfall rate (in centimeters per hour) at time t hours after midnight during a winter storm. Suppose

$$\int_2^6 r(t)\,dt = 24.$$

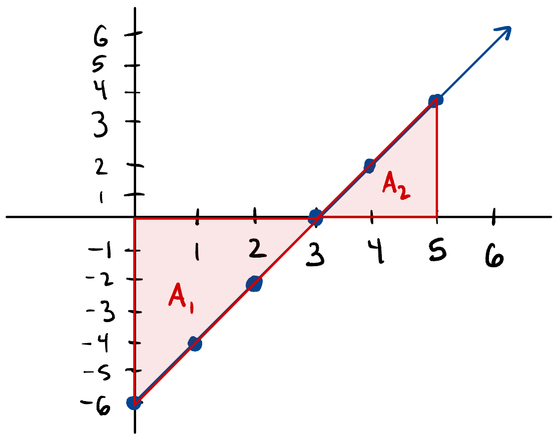
Which statement best describes this equation?

(a) At 2:00 am, snow is falling at a rate of 24 cm per hour.
(b) At 6:00 am, snow is falling at a rate of 24 cm per hour.
(c) A total of 24 centimeters of snow has fallen between 2:00 am and 6:00 am.
(d) The average snowfall rate between 2:00 am and 6:00 am is 24 cm per hour.
(e) At 6:00 am, a total of 24 centimeters of snow has fallen since midnight.

4. (4 points) A particle moves along a line so that its velocity (in meters per second) at time t (in seconds) is

$$v(t) = 2t - 6.$$

(a) Find the displacement of the particle during the time period $0 \le t \le 5$.

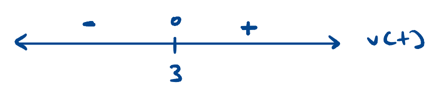


$$\text{displacement} = \int_0^5 v(t)\,dt$$
$$= \int_0^5 2t-6\,dt$$
$$= \left[t^2 - 6t\right]_0^5$$
$$= [25-30] - [0-0]$$
$$= -5 \text{ meters}$$

<u>ALTERNATE</u>: displacement $= -A_1 + A_2 = -\frac{1}{2}\cdot 3\cdot 6 + \frac{1}{2}\cdot 2\cdot 4 = -9+4 = -5$ meters

(b) Find the total distance traveled by the particle during this time period.

$v(t) = 0$ when $t = 3$



$$\text{distance traveled} = \int_0^5 \underbrace{|v(t)|}_{\text{speed}}\,dt$$
$$= \int_0^3 -v(t)\,dt + \int_3^5 v(t)\,dt$$
$$= \int_0^3 -2t+6\,dt + \int_3^5 2t-6\,dt$$
$$= \left[-t^2+6t\right]_0^3 + \left[t^2-6t\right]_3^5$$
$$= \Big([-9+18] - [0+0]\Big) + \Big([25-30] - [9-18]\Big)$$
$$= 9 + 4$$
$$= 13 \text{ meters}$$

<u>ALTERNATE</u>: distance traveled $= A_1 + A_2 = 9 + 4 = 13$ meters