Quiz 13

Format. This quiz has **3 multiple choice** question and **1 free-response** questions.

1. Compute an integral from a table.

Example: Use the information in the table to evaluate $\int_1^3 f'(z)\,dz$.

z	0	1	2	3	4	5
$f(z)$	3	2	3	8	7	10
$f'(z)$	-3	-5	-5	-2	0	1
$f''(z)$	0	-1	1	1	2	4

(a) -3
(b) 2
(c) 3
(d) 6
(e) -6

2. Compute an indefinite integral.

Example: Evaluate $\int \frac{1}{x}\,dx$.

(a) $\frac{1}{x} + C$
(b) $\ln|x| + C$
(c) $5x + C$
(d) $-5 + C$
(e) $x + C$

3. Interpret the meaning of an integral.

Example: Water starts to leak from a storage tank at 8:00 pm. If t is the number of minutes since 8:00 pm and $r(t)$ (in liters per minute) is the rate at which water leaks from the storage tank, what does the following represent?

$$\int_{10}^{20} r(t)\,dt = 50?$$

(a) Water leaks at a rate of 50 liters per minute from the storage tank at 8:20 pm.
(b) There are 50 liters of water left in the storage tank at 8:10 pm.
(c) 50 liters of water leak from the storage tank between 8:10 pm and 8:20 pm.
(d) Water leaks at a rate of 50 liters per minute from the storage tank at 8:10 pm.
(e) There are 50 liters of water left in the storage tank at 8:20 pm.

4. Given $v(t)$, compute the displacement and distance traveled.

Example:

A particle moves along a line so that its velocity (in m/s) at time t is

$$v(t) = t^2 - t - 6.$$

(a) Find the displacement of the particle during the time period $1 < t < 4$.

(b) Find the distance traveled during this time period.

Solution: By the Fundamental Theorem of Calculus,

$$\int_1^3 f'(z)\,dz = f(3) - f(1).$$

From the table, $f(3) = 8$ and $f(1) = 2$, so

$$\int_1^3 f'(z)\,dz = 8 - 2 = 6.$$

Thus the correct choice is **(d)** 6.

Solution: We use the basic antiderivative rule:

$$\int \frac{1}{x}\,dx = \ln|x| + C.$$

So the correct choice is **(b)** $\ln|x| + C$.

Solution: The rate $r(t)$ is measured in liters per minute, and t is minutes after 8:00 pm. The definite integral

$$\int_{10}^{20} r(t)\,dt$$

gives the *total amount of water* that leaks out between $t = 10$ and $t = 20$ minutes, i.e., between 8:10 pm and 8:20 pm. Since this integral equals 50, it means:

A total of 50 liters leak from the tank between 8:10 pm and 8:20 pm.

Thus the correct choice is **(c)**.

Solution:

(a) Displacement is the net change in position:

$$\text{displacement} = \int_1^4 v(t)\,dt = \int_1^4 (t^2 - t - 6)\,dt.$$

An antiderivative of $t^2 - t - 6$ is

$$\frac{1}{3}t^3 - \frac{1}{2}t^2 - 6t.$$

Thus

$$\begin{aligned}\int_1^4 (t^2 - t - 6)\,dt &= \left[\frac{1}{3}t^3 - \frac{1}{2}t^2 - 6t\right]_1^4 \\ &= \left(\frac{64}{3} - 8 - 24\right) - \left(\frac{1}{3} - \frac{1}{2} - 6\right) \\ &= -\frac{9}{2} \text{ meters.}\end{aligned}$$

The negative sign means the particle ends up 4.5 meters to the *left* of its position at $t = 1$.

(b) Distance is the integral of the speed $|v(t)|$:

$$\text{distance} = \int_1^4 |v(t)|\,dt.$$

First find where $v(t)$ changes sign on $[1, 4]$. Factor

$$v(t) = t^2 - t - 6 = (t - 3)(t + 2).$$

The roots are $t = -2$ and $t = 3$. On the interval $[1, 4]$, we have

$$v(t) < 0 \quad \text{for } 1 \le t < 3, \qquad v(t) > 0 \quad \text{for } 3 < t \le 4.$$

Therefore

$$\int_1^4 |v(t)|\,dt = \int_1^3 -v(t)\,dt + \int_3^4 v(t)\,dt$$

So

$$\text{distance} = \frac{22}{3} + \frac{17}{6} = \frac{44}{6} + \frac{17}{6} = \frac{61}{6} \text{ meters.}$$