Math 1300 **Name:** ______________________

Fall 2025

Quiz 11

1. (2 points) Evaluate $\lim_{x\to\infty} x\tan\frac{3}{x}$.

$$\begin{aligned}\lim_{x\to\infty} x\tan\frac{3}{x} &= \lim_{x\to\infty}\frac{\tan(3/x)}{1/x}\\ &\overset{\text{L'Hôpital}}{=} \lim_{x\to\infty}\frac{\sec^2(3/x)\cdot(-3/x^2)}{-1/x^2}\\ &= \lim_{x\to\infty} 3\sec^2(3/x)\\ &= \lim_{x\to\infty}\frac{3}{\cos^2(3/x)} = \boxed{3}.\end{aligned}$$

2. (2 points) Two positive numbers x and y have sum 12. Find the numbers that maximize $x^2 y$.

We are given $x + y = 12$ and asked to maximize x^2y. Since $y = 12 - x$, we can maximize:

$$f(x) = x^2(12 - x) = 12x^2 - x^3,$$

So

$$f'(x) = 24x - 3x^2 = 3x(8 - x).$$

There is one critical number at $x = 8$, since $x > 0$. This is a maximum (why?). Hence $y = 4$.

$$\boxed{x = 8,\ y = 4}, \quad \text{max value is } 256.$$

3. (2 points) Let $f(x) = 6x^2 - 4x + 3$. Find the antiderivative of $f(x)$ that passes through the point $(1, 5)$.

$$F'(x) = f(x) \quad\Rightarrow\quad F(x) = 2x^3 - 2x^2 + 3x + C.$$

Use $F(1) = 5$: $2 - 2 + 3 + C = 5 \ \Rightarrow\ C = 2$. Hence

$$\boxed{F(x) = 2x^3 - 2x^2 + 3x + 2}.$$

4. (4 points) A bakery sells 100 loaves of sourdough per day at \$8 each. A survey shows that for every \$1 decrease in price, daily sales increase by 10 loaves. What **price** maximizes the bakery's daily revenue? Be sure to justify that the value you found gives the absolute maximum.

1. **Demand function.** Each \$1 decrease adds 10 loaves:

x	$p(x)$
100	8
110	7

Slope is $m = \dfrac{\Delta p}{\Delta x} = -\dfrac{1}{10}$.

$$p(x) = -\frac{1}{10}x + b, \quad 8 = -\frac{1}{10} \cdot 100 + b \Rightarrow b = 18.$$

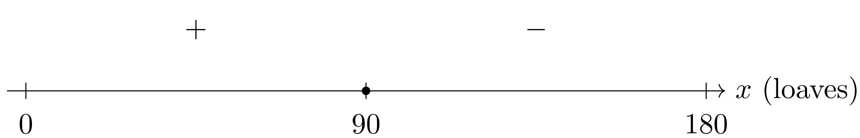
$$\boxed{p(x) = 18 - \frac{x}{10}}$$

2. **Revenue function.**

$$R(x) = x\,p(x) = x\left(18 - \frac{x}{10}\right) = 18x - \frac{x^2}{10}$$

3. **Optimize revenue.**

$$R'(x) = 18 - \frac{x}{5}, \qquad R'(x) = 0 \implies x = 90 \text{ (only critical number).}$$



Since $R'(x) > 0$ on $(\infty, 90)$ and $R'(x) < 0$ on $(90, \infty)$, R increases up to $x = 90$ and decreases after. Therefore, by the First Derivative Test for Absolute Extreme Values, R attains its *absolute maximum* at $x = 90$.

4. **Price.**

$$p(90) = 18 - \frac{90}{10} = 9 \qquad \Longrightarrow \qquad \boxed{\text{Price that maximizes revenue } = \text{ \$9}}$$