Quiz 11 Outline

Format. This quiz has **3 short-answer** questions and **1 free-response** question.

1. Use l'Hôpital's rule to evaluate a limit (indeterminate difference, product, or power).

> **Example:** Evaluate $\lim_{x \to 0^+} x \ln x$.
>
> **Answer:**

2. Optimizing a quantity subject to a constraint.

> **Example:** Let x and y be positive numbers subject to the constraint $xy = 12$. What values of x and y *minimize* the expression $S = x + 3y$?
>
> **Answer:**

3. Find an antiderivative passing through a point.

> **Example:** Let $f(x) = 3x^2 - 4x + 1$. Find the antiderivative $F(x)$ of $f(x)$ that passes through the point $(0, 2)$.
>
> **Answer:**

4. Find a demand function and use it to maximize a revenue function $R(x)$.

Example: A store has been selling 200 computers per week at \$350 each. A market survey indicates that for each decrease in price of \$10, the number of units sold will increase by 20 per week. What price will maximize the store's weekly revenue?

Solution: Rewrite as a quotient and apply L'Hôpital's Rule:

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+}(-x) = \boxed{0}.$$

Solution: With the constraint $xy = 12$ and $x > 0$, write $y = \dfrac{12}{x}$. Then

$$S(x) = x + 3y = x + \frac{36}{x}, \qquad x > 0.$$

Differentiate:

$$S'(x) = 1 - \frac{36}{x^2}.$$

Critical point from $S'(x) = 0$: $x^2 = 36 \Rightarrow x = 6$. For $0 < x < 6$, $S'(x) < 0$ so S is decreasing; for $x > 6$, $S'(x) > 0$ so S is increasing. Hence $x = 6$ gives a minimum. By the **FDTAEV**, this is the *absolute* minimum since $x = 6$ is the only sign change in S' on the open interval $(0, \infty)$.

Thus $x = 6$, $y = \dfrac{12}{6} = 2$, and

$$S_{\text{min}} = 6 + 3 \cdot 2 = 12.$$

Solution: Antidifferentiating the function

$$F'(x) = f(x) = 3x^2 - 4x + 1$$

We get

$$F(x) = x^3 - 2x^2 + x + C.$$

Now use $F(0) = 2$ to find $C = 2$. Hence

$$\boxed{F(x) = x^3 - 2x^2 + x + 2}.$$

Solution: Let x denote the number of computers sold per week and $p(x)$ the selling price (in dollars).

(1) Demand function. The survey says every +20 units in x lowers price by \$10:

x	$p(x)$
200	350
220	340
240	330
260	320
300	300

Therefore,

$$m = \frac{\Delta p}{\Delta x} = \frac{-10}{20} = -\frac{1}{2}.$$

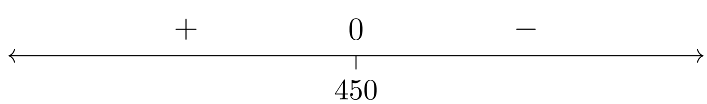
Using the point $(200, 350)$ and point-slope form,

$$p - 350 = -\tfrac{1}{2}(x - 200) \quad \Longrightarrow \quad p(x) = 450 - \tfrac{1}{2}x.$$

(2) Maximize revenue on the closed domain. Revenue is $R(x) = x\,p(x)$:

$$R(x) = x\big(450 - \tfrac{1}{2}x\big) = 450x - \tfrac{1}{2}x^2.$$

Critical numbers come from $R'(x) = 450 - x$. $R'(x) = 0$ when $x = 450$, so we have one critical number.



Thus R increases on $x < 450$ and decreases on $x > 450$, so $x = 450$ gives the *absolute* maximum of $R(x)$ by the *F.D.T.A.E.V.*

(3) Optimal price and maximum revenue.

$$p(450) = 450 - \tfrac{1}{2}(450) = 225, \qquad R(450) = 450 \cdot 225 = \$101{,}250.$$

Conclusion. The store maximizes weekly revenue by selling $\boxed{450}$ units at $\boxed{\$225}$ each. The maximum revenue is $\boxed{\$101{,}250}$.