Math 1300
Fall 2025
Quiz 10

Name: Champ

1. (2 points) Evaluate the limit $\lim_{x\to 0} \dfrac{e^x - 1 - x - \frac{x^2}{2}}{x^2}$. $\frac{0}{0}$

   (a) 0
   (b) $\frac{1}{2}$
   (c) $\frac{1}{6}$
   (d) 1
   (e) Does not exist

$$= \lim_{x\to 0} \frac{e^x - 1 - x}{2x} \quad \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{e^x - 1}{2} = 0$$

2. (2 points) Let $f(x) = \dfrac{x^2}{x-4}$. Find all critical numbers of $f(x)$.

$$f'(x) = \frac{(x-4)\cdot 2x - x^2 \cdot (1)}{(x-4)^2}$$

$$= \frac{2x^2 - 8x - x^2}{(x-4)^2}$$

$$= \frac{x^2 - 8x}{(x-4)^2}$$

$$= \frac{x(x-8)}{(x-4)^2}$$

!!

$x = 4$ is NOT a critical number because it is not in the domain of $f(x)$

Answer: $x = 0$, $x = 8$

3. (2 points) Let $f(x) = x^4 - 6x^2$. Find the x-value(s) of all inflection points of f.

$$f'(x) = 4x^3 - 12x$$

$$f''(x) = 12x^2 - 12 = 12(x^2-1) = 12(x-1)(x+1)$$

	12	$x-1$	$x+1$	f''
$x<-1$	+	−	−	+
$-1<x<1$	+	−	+	−
$x>1$	+	+	+	+

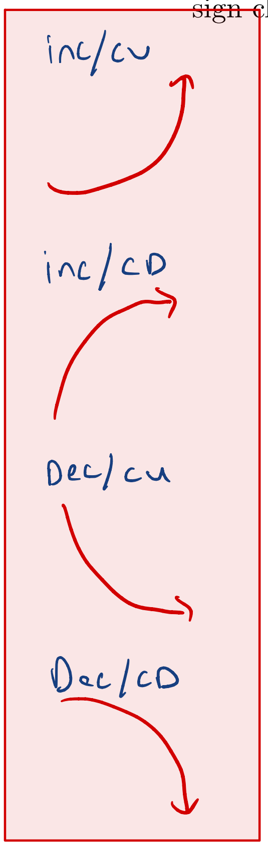
Answer: $x = -1$, $x = 1$

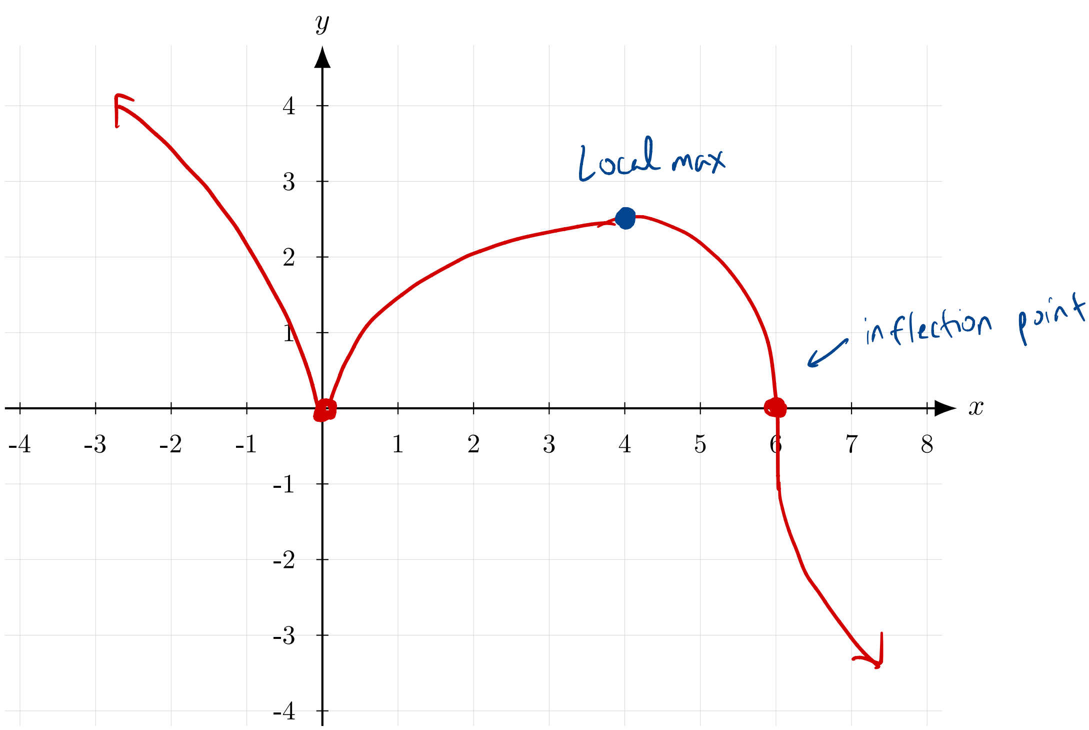
4. (4 points) Consider the function $f(x)$ and its derivatives:

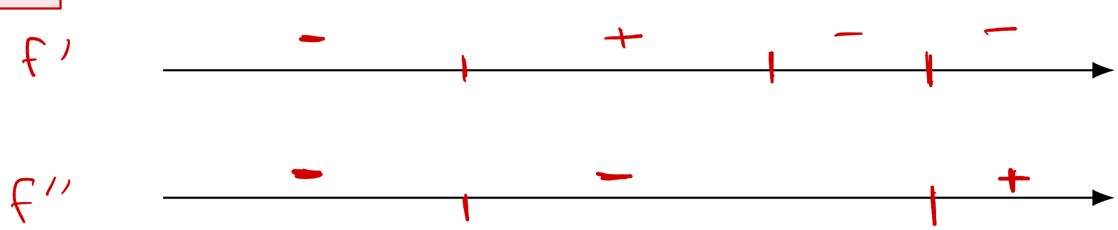
$$f(x) = x^{2/3}(6-x)^{1/3}, \qquad f'(x) = \frac{4-x}{x^{1/3}(6-x)^{2/3}}, \qquad f''(x) = \frac{-8}{x^{4/3}(6-x)^{5/3}}.$$

0 @ 0, 6 0 @ 4, DNE @ 0, 6 DNE @ 0, 6

Graph $f(x)$ as accurately as possible. Be sure to consider where the function is increasing, decreasing, concave up, or concave down. Also be sure to include any local extrema and inflection points. Number lines have been included below the graph for your convenience if you would like to use them to make sign charts.







Increasing / Decreasing

	$4-x$	$x^{1/3}$	$(6-x)^{2/3}$	f'	
$x<0$	+	−	+	−	> min
$0<x<4$	+	+	+	+	> max
$4<x<6$	−	+	+	−	> neither
$x>6$	−	+	+	−	

Concavity

	-8	$x^{4/3}$	$(6-x)^{5/3}$	f''	
$x<0$	−	+	+	−	
$0<x<6$	−	+	+	−	> infl. point
$x>6$	−	+	−	+	