Quiz 10 Outline

Format. This quiz has **1 multiple choice** question, **2 short-answer choice** questions, and **1 free-response** question.

1. Use l'Hôpital's rule to evaluate a limit.

Example: Evaluate the following limit:

$$\lim_{x \to 0} \frac{\ln(1+x) - x + \frac{1}{2}x^2}{x^3}.$$

(a) The limit does not exist.

(b) The limit is equal to $\frac{1}{3}$.

(c) The limit is equal to 0.

(d) The limit is equal to $-\frac{1}{3}$.

(e) The limit is equal to $\frac{1}{2}$.

2. Compute the critical numbers of a function.

Example: Let $f(x) = \dfrac{x^2}{x+2}$. Find all critical numbers of $f(x)$.

Answer:

3. Find the x-values of the inflection points of a function.

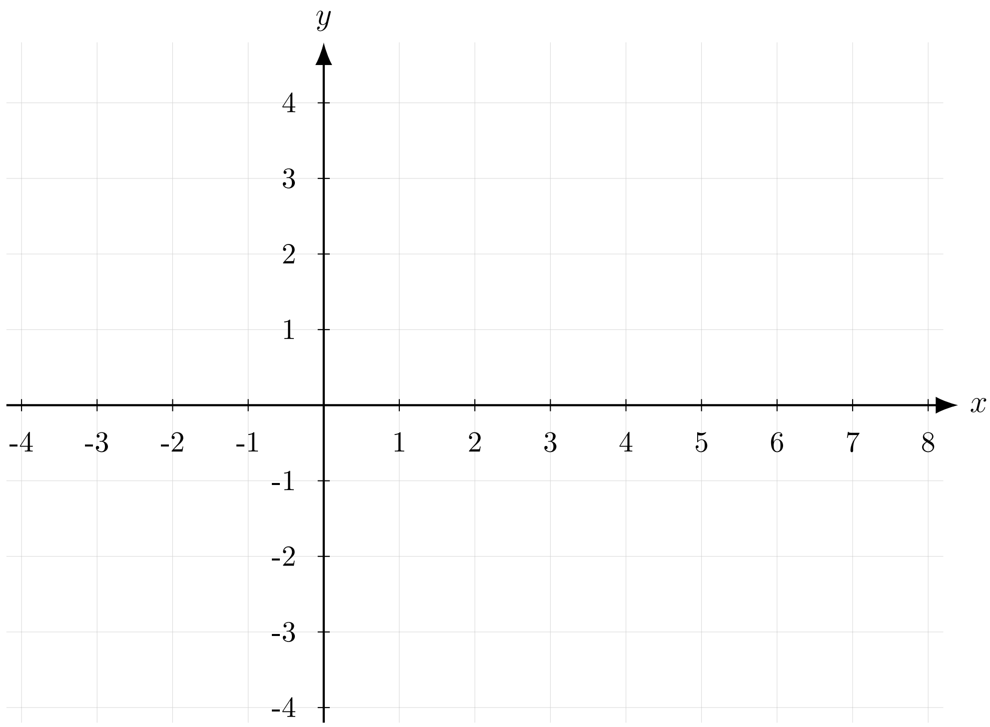
Example: Let $f(x) = x^5 + 5x^4$. Find the x-value(s) for all inflection points of $f(x)$.

Answer:

4. Consider the function $f(x)$ and its derivatives:

$$f(x) = x^{2/3}(6-x)^{1/3}, \qquad f'(x) = \frac{4-x}{x^{1/3}(6-x)^{2/3}}, \qquad f''(x) = \frac{-8}{x^{4/3}(6-x)^{5/3}}.$$

Graph $f(x)$ as accurately as possible. Be sure to consider where the function is increasing, decreasing, concave up, or concave down. Also be sure to include any local extrema and inflection points. Number lines have been included below the graph for your convenience if you would like to use them to make sign charts.



Solution: $\lim_{x\to 0} \dfrac{\ln(1+x) - x + \frac{1}{2}x^2}{x^3}$ is $0/0$. Apply L'Hôpital three times:

$$\lim_{x\to 0} \frac{\frac{1}{1+x} - 1 + x}{3x^2} = \lim_{x\to 0} \frac{-\frac{1}{(1+x)^2} + 1}{6x} = \lim_{x\to 0} \frac{\frac{2}{(1+x)^3}}{6} = \frac{2}{6} = \boxed{\frac{1}{3}}.$$

Solution: Consider $f(x) = \dfrac{x^2}{x+2}$, where the domain is $x \neq -2$.

$$f'(x) = \frac{(2x)(x+2) - x^2}{(x+2)^2} = \frac{x^2 + 4x}{(x+2)^2} = \frac{x(x+4)}{(x+2)^2}.$$

Critical numbers occur where $f'(x) = 0$ (and x is in the domain) or where f' is undefined but f is defined. From $x(x+4) = 0$, we get $x = 0$ and $x = -4$, both in the domain. At $x = -2$, f is undefined, so it is not a critical number.

$$\boxed{\text{Critical numbers: } -4,\ 0.}$$

Solution: $f(x) = x^5 + 5x^4$. Then

$$f'(x) = 5x^4 + 20x^3, \qquad f''(x) = 20x^2(x+3).$$

Candidates for inflection points satisfy $f''(x) = 0$: $x = 0, -3$. Since $20x^2 \geq 0$, the sign of f'' is the sign of $x + 3$: it changes at $x = -3$ (from < 0 to > 0) but not at $x = 0$. Hence the only inflection x-value is

$$\boxed{-3}.$$

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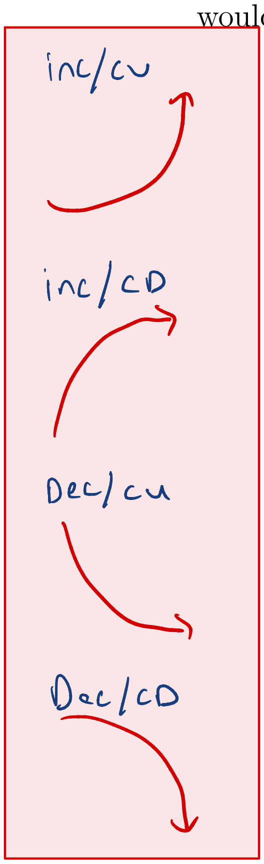

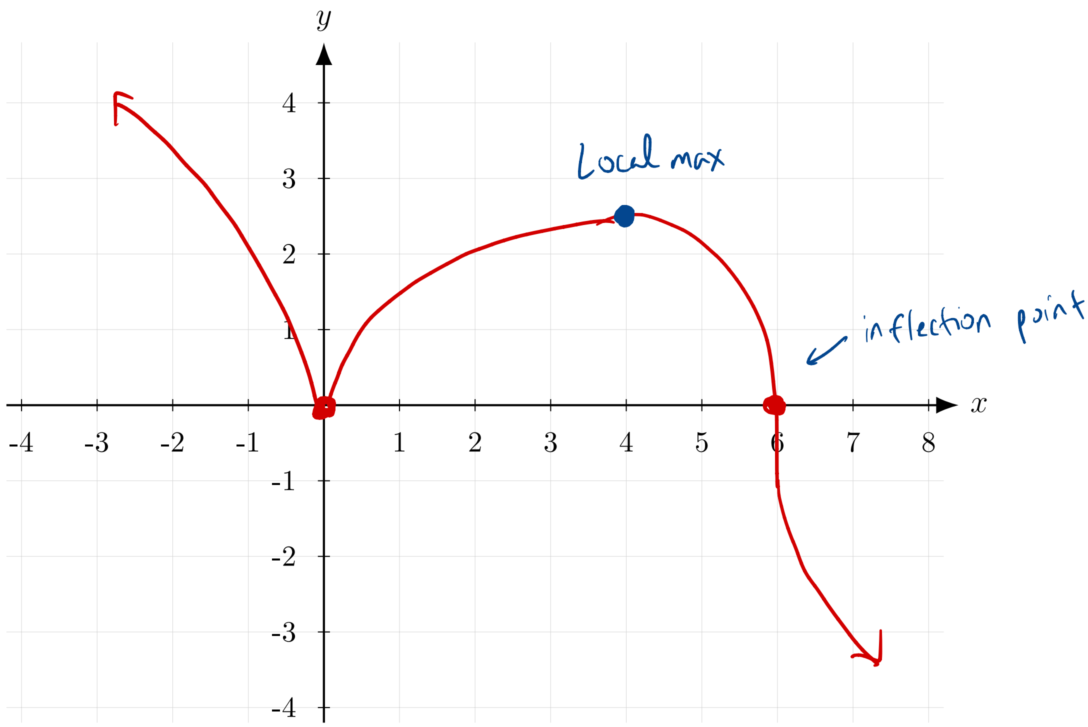

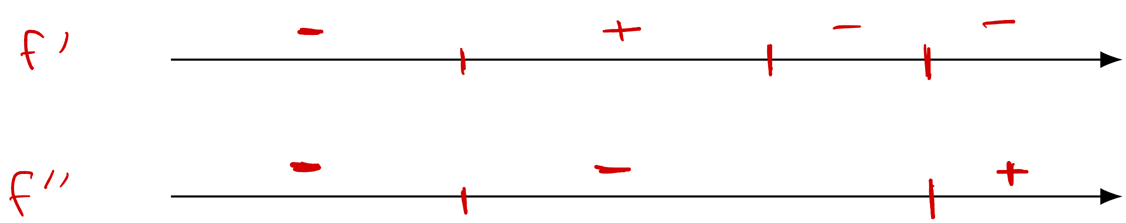

Increasing / Decreasing

	$4-x$	$x^{1/3}$	$(6-x)^{2/3}$	f'	
$x<0$	+	−	+	−	} min
$0<x<4$	+	+	+	+	} max
$4<x<6$	−	+	+	−	} neither
$x>6$	−	+	+	−	

Concavity

	-8	$x^{4/3}$	$(6-x)^{5/3}$	f''	
$x<0$	−	+	+	−	
$0<x<6$	−	+	+	−	} infl. point
$x>6$	−	+	−	+	