

Linear Approximation

1. Let $f(x) = \ln(1 + 2x)$ and $a = 0$.

(a) Find the linearization $L(x)$ at $x = 0$.

(b) Use $L(x)$ to approximate $\ln(1.06)$. State the x you plug into L .

(c) Is $L(x)$ an overestimate or an underestimate?

2. Let $f(x) = \sqrt[3]{x}$ and $a = 8$.

(a) Find $L(x)$ at $a = 8$.

(b) Use L to approximate $\sqrt[3]{7.7}$ and $\sqrt[3]{8.2}$. Are these approximations overestimates or underestimates?

3. Let $f(x) = \frac{1}{x^2 + 5}$ and $a = 1$.

(a) Find $L(x)$ at $x = 1$.

(b) Use L to approximate $f(1.1)$ and $f(0.9)$ and decide whether L is an overestimate or an underestimate near $a = 1$.

4. Let $f(x) = \sqrt{1 + x}$ and $a = 0$.

(a) Find $L(x)$.

(b) Use L to estimate $\sqrt{1.04}$ and decide if your estimate is an overestimate or an underestimate.

5. Below are four functions and four linearizations at $a = 0$. Match each function with its correct $L(x)$ and justify each match by computing $f(0)$ and $f'(0)$.

(I) $f(x) = \sqrt{4 + x}$ at $a = 0$

(A) $L(x) = 2x$

(II) $f(x) = \frac{1}{3 + x}$ at $a = 0$

(B) $L(x) = 1 + x$

(III) $f(x) = \sin(2x)$ at $a = 0$

(C) $L(x) = 2 + \frac{1}{4}x$

(IV) $f(x) = e^x$ at $a = 0$

(D) $L(x) = \frac{1}{3} - \frac{1}{9}x$