Midterm 1 Study Guide

MATH1300 - Calculus I

Fall 2025

Contents

1.1-1.3 Precalculus Review

Lines

1. Find the equation of the line parallel to $y = -3x + 7$ that passes through $(2, 4)$.
2. Find the equation of the line perpendicular to $y = \frac{1}{2}x - 5$ that passes through $(0, 6)$.
3. Write the equation of the line through $(-1, 2)$ and $(3, -4)$ in slope-intercept form.
4. A line passes through $(5, 1)$ with slope -2. Write its equation in point-slope and slope-intercept form.

Polynomial End Behavior

5. Determine the end behavior of
$$f(x) = -4x^5 + 2x^2 - 7.$$
6. Determine the end behavior of
$$f(x) = 3x^4 - 5x^3 + x - 1.$$
7. Determine the end behavior of
$$f(x) = -x^{10} + 8x^7 - 2.$$

Logs & Exponentials

8. Evaluate $\log_5(125)$.
9. Simplify $\log_2(16) - \log_2(4)$.
10. Solve for x: $3^x = 81$.

Difference Quotient Practice

11. For $f(x) = x^2 + 3x$, compute
$$\frac{f(a+h) - f(a)}{h}$$
and simplify.
12. For $f(x) = \sqrt{x+1}$, compute
$$\frac{f(2+h) - f(2)}{h}$$
and simplify.

Polynomial Algebra

13. Write $P(x) = -6x^2 + 4x - 7$ in standard form. State its degree, coefficients, leading coefficient, and terms.
14. For $P(x) = 3x^4 - 2x^3 + x - 5$, identify degree, leading coefficient, and constant term.
15. For $Q(x) = (x-1)(x+2)(x-3)$, expand into standard form and state its degree and leading coefficient.

Polynomial Equations & Factoring

16. Solve $x^2 - 5x + 6 = 0$ by factoring.
17. Solve $2x^2 + 7x + 3 = 0$.
18. Factor $x^3 - 27$ completely.

Trig Review

19. Evaluate $\sin\frac{\pi}{6}$, $\cos\frac{\pi}{3}$, and $\tan\frac{\pi}{4}$.
20. Evaluate $\csc\frac{\pi}{2}$, $\sec 0$, and $\cot\frac{\pi}{3}$.
21. Find the amplitude and period of
$$y = 3\sin(2x).$$
22. Find the amplitude and period of
$$y = -\tfrac{1}{2}\cos(\pi x).$$

2.1 Tangent & Velocity Problems

Average Velocity (Data Tables)

1. A particle's position (m) at times t (s):

t	0	1	2	3	4
$s(t)$	0	2	7	15	26

   Compute the average velocity on $[1, 3]$ and $[2, 4]$. Include units.

2. A ball's height $s(t)$ (m):

t	0	1	2	3	4
$s(t)$	20	15	11	8	6

   Compute the average velocity on $[0, 2]$ and $[2, 4]$. Include units.

Average Velocity (Formulas)

3. $s(t) = t^2 + 3t$ (m). Compute the average velocity on $[2, 5]$ and $[5, 6]$.

4. $s(t) = \sqrt{t + 4}$ (m). Compute the average velocity on $[0, 1]$ and $[1, 4]$.

Secant Slopes and Tangent Slope

5. For $f(x) = x^2$, find the slope of the line between $(2, f(2))$ and $(2 + h, f(2 + h))$ for the following values of h:

h	1	0.5	0.1	0.01	-0.01	-0.1
slope						

   Estimate the tangent slope at $x = 2$.

Average Rates from Real-World Data

6. A car travels 150 miles in 3 hours, then 60 miles in the next hour. Compute average speed on $[0, 3]$, on $[3, 4]$, and on $[0, 4]$.

7. $s(t) = 100t - 5t^2$ (m). Compute average velocity on $[2, 3]$ and $[2, 2.1]$. What does this suggest about instantaneous velocity at $t = 2$?

8. A turkey cools on a counter. Its temperature T (°F) is measured at times t (min):

t	50	55	60	65	70	75
T	120	114	109	105	101	98

   Estimate the instantaneous cooling rate at $t = 60$ by taking the average of the secant slopes on $[55, 60]$ and $[60, 65]$ (include units).

9. The depth of water in a reservoir $W(x)$ (m) is measured over 365 days. Suppose $W(150) = 50$, $W(200) = 52$, $W(250) = 54$, $W(300) = 49$. Estimate the average rate of change on $[150, 250]$ and interpret its meaning.

2.2 Limits

One-Sided Limits

1. Compute $\lim_{x\to 0^-} \frac{1}{x}$ and $\lim_{x\to 0^+} \frac{1}{x}$.

2. Compute $\lim_{x\to (\pi/2)^-} \tan x$ and $\lim_{x\to (\pi/2)^+} \tan x$.

Absolute Value Limits

3. Compute $\lim_{x\to 0^-} \frac{|x|}{x}$ and $\lim_{x\to 0^+} \frac{|x|}{x}$.

4. Compute $\lim_{x\to 2^-} \frac{|x-2|}{x-2}$ and $\lim_{x\to 2^+} \frac{|x-2|}{x-2}$.

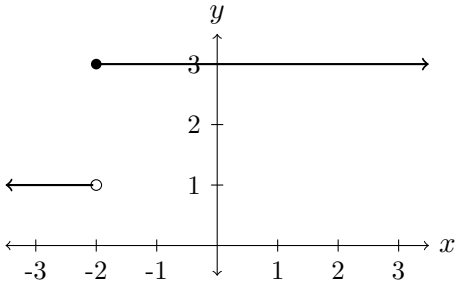
Piecewise Functions

5. Let
$$f(x) = \begin{cases} 2x+1 & x < 1 \\ 5 & x = 1 \\ x^2 & x > 1 \end{cases}$$
Evaluate $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1^+} f(x)$, and $\lim_{x\to 1} f(x)$.

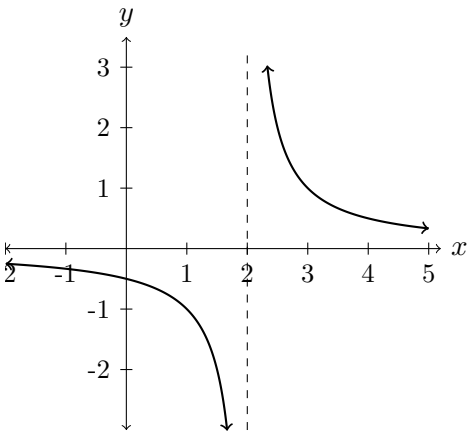
6. Let
$$g(x) = \begin{cases} x^2 & x \le 0 \\ \sqrt{x} & x > 0 \end{cases}$$
Compute $\lim_{x\to 0^-} g(x)$, $\lim_{x\to 0^+} g(x)$, and $\lim_{x\to 0} g(x)$.

Graph-Based Interpretation

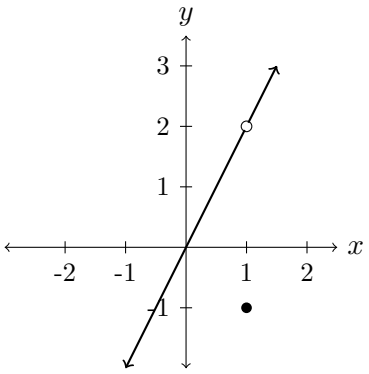
7. From the graph of f below, find $\lim_{x\to -2^-} f(x)$, $\lim_{x\to -2^+} f(x)$, $\lim_{x\to -2} f(x)$, and $f(-2)$.



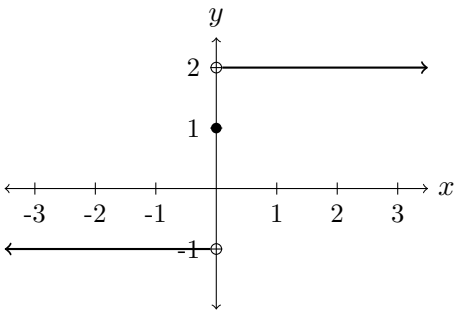
8. From the graph of f below, find $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$. Does $\lim_{x\to 2} f(x)$ exist?



9. From the graph of f below, find $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1^+} f(x)$, $\lim_{x\to 1} f(x)$, and $f(1)$.



10. From the graph of f below, find $\lim_{x\to 0^-} f(x)$, $\lim_{x\to 0^+} f(x)$, $\lim_{x\to 0} f(x)$, and $f(0)$.



2.3 Limit Laws

Direct Use of Limit Laws

1. Compute $\lim_{x\to 2}(3x^2 - 4x + 7)$.

2. Compute $\lim_{x\to -1}(x^3 + 2x^2 - 5x)$.

3. Compute $\lim_{x\to 4}(x^2 + \sqrt{x})$.

Rational Functions

4. Compute $\lim_{x\to 3}\dfrac{x^2 - 9}{x - 3}$.

5. Compute $\lim_{x\to -2}\dfrac{x^3 + 8}{x + 2}$.

Roots

6. Compute $\lim_{x\to 15}\sqrt[4]{x+1}$.

7. Compute $\lim_{x\to 0}\dfrac{\sqrt{x+4} - 2}{x}$.

8. Compute $\lim_{x\to 0}\dfrac{\sqrt{1+x} - \sqrt{1-x}}{x}$.

9. Compute $\lim_{x\to 1}\dfrac{\sqrt{5x+4} - 3}{x - 1}$.

Squeeze Theorem

10. Suppose h is a function such that

$$x - 1 \;\le\; h(x) \;\le\; x^2 + x - 2.$$

for all real x. Evaluate $\lim_{x\to 1} h(x)$, if it exists.

11. Evaluate $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$.

12. Exponential sandwich: For $x \neq 0$,

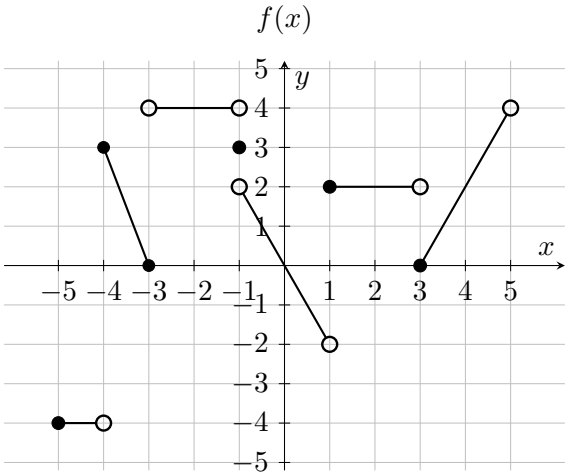
$$-x^2 \le x^2 e^{-\frac{1}{x^2}} \sin\left(\tfrac{1}{x}\right) \le x^2.$$

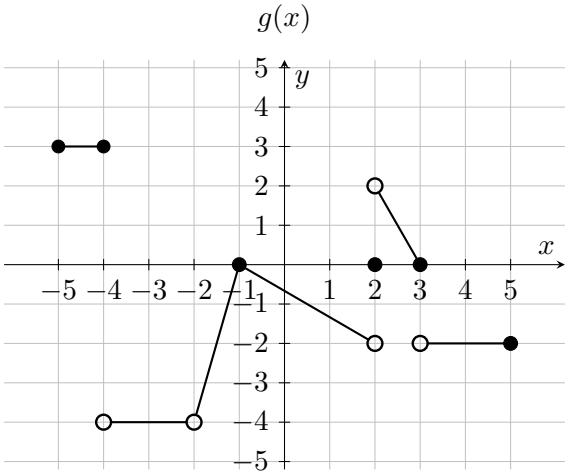
Use this to evaluate

$$\lim_{x\to 0} x^2 e^{-\frac{1}{x^2}} \sin\left(\tfrac{1}{x}\right).$$

Limits from Graphs

13. Use the graphs of $f(x)$ and $g(x)$ to answer the questions below.





(a) Compute $\lim_{x\to 5^-}\big(f + g\big)$

(b) Compute $\lim_{x\to 3}\big(f - g\big)$

(c) Compute $\lim_{x\to 1} f^2$

(d) Compute $\lim_{x\to -2}\big(5g\big)$

(e) Compute $\lim_{x\to 0}\left(\dfrac{g}{f}\right)$

(f) Compute $\lim_{x\to -1}\left(\dfrac{f}{g}\right)$

(g) Compute $\lim_{x\to -4}\big(fg\big)$

2.5 Continuity & IVT

Continuity at a Point

1. Determine whether $f(x) = \dfrac{x^2 - 4}{x - 2}$ is continuous at $x = 2$.

2. Determine whether $f(x) = \tan x$ is continuous at $x = \pi/2$.

Piecewise Continuity

3. Let
$$f(x) = \begin{cases} x^2 + 1 & x < 2 \\ ax + b & x \geq 2 \end{cases}$$
Find a, b so that f is continuous at $x = 2$.

4. Let
$$g(x) = \begin{cases} \sqrt{x+1} & x > 0 \\ k & x = 0 \\ x^2 + 1 & x < 0 \end{cases}$$
Find k so that g is continuous at $x = 0$.

5. Let
$$F(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3}, & x \neq 3, \\ c, & x = 3 \end{cases}$$
Find c so that F is continuous at $x = 3$.

6. Let
$$G(x) = \begin{cases} \dfrac{\sqrt{x-1} - 2}{x - 5}, & x \neq 5, \\ c, & x = 5. \end{cases}$$
Find c so that G is continuous at $x = 5$.

Classifying Discontinuities

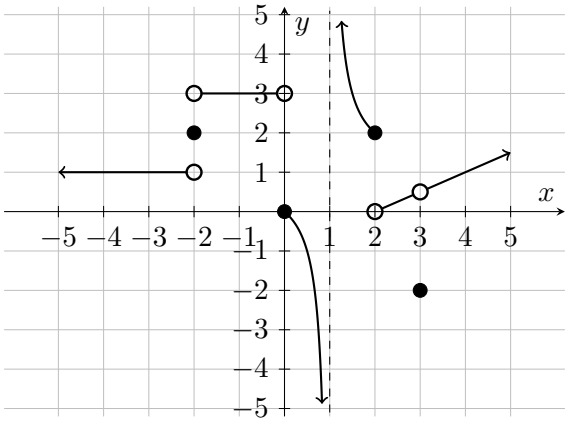
7. Classify the discontinuity of $f(x) = \dfrac{x^2 - 1}{x - 1}$ at $x = 1$.

8. Classify the discontinuity of $f(x) = \dfrac{1}{x^2}$ at $x = 0$.

9. Classify the discontinuity of $f(x) = \dfrac{|x|}{x}$ at $x = 0$.

Graph-Based Reasoning

10. From the graph of f below, determine all x-values where f is discontinuous. Classify each such x-value as a *removable*, *jump*, or *infinite* discontinuity.



Intermediate Value Theorem

11. $f(x) = x^3 - 5x + 2$ is continuous. Show there is a solution to $f(x) = 0$ in $(0, 1)$.

12. Prove there exists $0 < x < 1$ such that $\cos x = x$.

13. Let $f(x) = e^x - 4$. Show that $f(x) = 0$ has a solution in $(1, 2)$.

Intervals of Continuity

14. Let
$$h(x) = \begin{cases} \dfrac{1}{x+2} & x < -2, \\ x^2 - 1 & -2 \leq x < 1, \\ \sqrt{x-1} & x \geq 1. \end{cases}$$
List all intervals on which h is continuous. Classify the behavior at $x = -2$ and $x = 1$.

15. Let $p(x) = \ln(x - 3) + \dfrac{x}{x - 5}$. State the domain and the intervals on which p is continuous. Identify points of discontinuity and their types.

Continuity Theorems & Compositions

16. Suppose f is continuous at $x = 0$ and g is continuous at $x = 4$. Moreover, suppose $f(0) = 4$, and $\lim_{x\to 0} g(f(x)) = 7$. Find $g(4)$ and justify.

17. Let $h(x) = \sqrt{f(x)}$ and suppose f is continuous at a with $f(a) = 9$. Explain why h is continuous at a and compute $\lim_{x\to a} h(x)$.

Repairing Removable Discontinuities

18. For $f(x) = \dfrac{x^2 - 1}{x - 1}$, define a function g that agrees with f for $x \neq 1$ and is continuous at $x = 1$. What is $g(1)$?

19. For $f(x) = \dfrac{x^3 - 8}{x - 2}$, construct g that removes the discontinuity at $x = 2$ and state $g(2)$.

2.6 Limits at Infinity & Asymptotes

Horizontal Asymptotes (Rational)

1. Find the horizontal asymptote(s) of

$$f(x) = \frac{3x^2 + 5}{2x^2 - 7}.$$

2. Find the horizontal asymptote(s) of

$$f(x) = \frac{5x^3 - 4}{x^2 + 1}.$$

3. Compute the following limits:

$$\lim_{x\to\infty} \frac{2x+1}{x^3+4} \quad \text{and} \quad \lim_{x\to-\infty} \frac{2x+1}{x^3+4}$$

4. Determine the end behavior of

$$f(x) = \frac{-4x^5 + 3}{2x^5 - 7x^2 + 1}.$$

Vertical Asymptotes (Rational)

5. Find the vertical asymptote(s) of

$$f(x) = \frac{x-1}{x^2-9}.$$

6. Find the vertical asymptote(s) of

$$f(x) = \frac{1}{(x-2)^2}.$$

7. Find the vertical asymptote(s) of

$$f(x) = \frac{1}{x(x-1)}.$$

Vertical Asymptotes (Log/Trig)

8. Find the vertical asymptote(s) of

$$f(x) = \ln(x).$$

9. Find the vertical asymptote(s) of

$$f(x) = \tan x$$

in $(-\pi, \pi)$.

Oblique (Slant) Asymptotes

10. Find the slant asymptote of

$$f(x) = \frac{2x^2 - 3x + 1}{x - 1}.$$

Then list any vertical asymptotes.

11. Determine the slant asymptote of

$$f(x) = \frac{x^3 + x^2 - 1}{x^2 - 1}$$

and identify all vertical asymptotes.

Horizontal Asymptotes (Non-Rational)

12. Determine the horizontal asymptotes of $f(x) = \arctan x$.

13. Find the horizontal asymptotes of

$$f(x) = \frac{1}{1 + e^{-x}}$$

and justify using limits as $x \to \pm\infty$.

14. Evaluate

$$\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right) \quad \text{and} \quad \lim_{x\to-\infty} \left(\sqrt{x^2+1} - x\right).$$

Interpret the results in terms of asymptotes/end behavior.

15. Compute $\lim_{x\to\infty} \left(x - \sqrt{x^2 + 2x}\right)$ and decide whether $y = 0$ is a horizontal asymptote.

Growth-Rate Comparisons (Poly vs. Exp vs. Log)

16. Compute $\lim_{x\to\infty} \frac{\ln x}{x}$ and $\lim_{x\to\infty} \frac{x}{\ln x}$.

17. Compute $\lim_{x\to\infty} \frac{e^x}{x^5}$ and $\lim_{x\to\infty} \frac{x^7}{e^{0.1x}}$.

18. Decide: $\lim_{x\to\infty} \frac{x^3 \ln x}{e^{\sqrt{x}}}$ (explain which term dominates).

19. Determine $\lim_{x\to\infty} \frac{(\ln x)^4}{x^{1/3}}$ and interpret (which grows faster?).

Vertical Asymptotes (One-Sided Sign Analysis)

20. Evaluate $\lim_{x\to -6^-} \dfrac{x+5}{x+6}$ and $\lim_{x\to -6^+} \dfrac{x+5}{x+6}$; conclude whether $\lim_{x\to -6}$ exists and classify the behavior.

21. Compute $\lim_{x\to 2^-} \dfrac{1}{x-2}$ and $\lim_{x\to 2^+} \dfrac{1}{x-2}$; compare with $\lim_{x\to 2} \dfrac{1}{(x-2)^2}$.

22. Determine the one-sided limits of

$$f(x) = \frac{2x-1}{(x+3)^2}$$

as $x \to -3^{\pm}$. State the vertical asymptote and sign on each side.

Holes vs. Vertical Asymptotes

23. Find the vertical asymptote(s) of

$$f(x) = \frac{x^2+2x-3}{x^2-9}.$$

24. Identify all vertical asymptotes of

$$f(x) = \frac{x^2-1}{x-1}.$$

If none, explain why and describe the point removed from the domain.

25. For $f(x) = \dfrac{x^2-4}{x^2-x-2}$, factor and cancel where possible. List vertical asymptotes and any holes.

2.7 Derivatives & Rates of Change

Definition of the Derivative

1. Use the limit definition to compute $f'(2)$ for $f(x) = x^2$.
2. Use the limit definition to compute $f'(0)$ for $f(x) = x^3$.
3. Use the limit definition to compute $f'(a)$ for $f(x) = mx + b$ (verify result).

Radicals / Rationals

4. Use the limit definition to compute $f'(1)$ for $f(x) = \sqrt{x}$.
5. Use the limit definition to compute $f'(3)$ for $f(x) = \frac{1}{x}$.
6. Use the limit definition to compute $f'(4)$ for $f(x) = \sqrt{x+1}$.

Tangent Lines at a Point

7. For the curve $y = 6x - x^2$, verify the point $(2, 8)$ lies on the curve, find the slope of the tangent at $x = 2$, and write the tangent line at $(2, 8)$.
8. Let $y = 1 + 5x^2$.
   (a) Find the slope of the tangent at a general point $x = a$ (simplify).
   (b) Find the tangent line at $(1, 6)$.
9. Find the tangent line to $y = \sqrt{x+4}$ at $x = 5$ (give point–slope and slope–intercept forms).
10. The tangent to $y = f(x)$ at $(4, -1)$ passes through $(0, -5)$. Find $f(4)$ and $f'(4)$.
11. The tangent to $y = f(x)$ at $(1, 2)$ passes through $(-3, -6)$. Find $f(1)$ and $f'(1)$.

Recognize "Derivative from a Limit"

12. Identify f and a so that
$$\lim_{h\to 0} \frac{\big(2(3+h)^2 - 5\big) - 13}{h} = f'(a).$$
13. Identify f and a so that
$$\lim_{h\to 0} \frac{\sqrt{9+h} - 3}{h} = f'(a).$$
14. Identify f and a so that
$$\lim_{h\to 0} \frac{\ln(1+h) - \ln 1}{h} = f'(a).$$