Derivatives Reference Sheet

Basic Rules & Combinations of Functions

Function	Derivative	
x^n	$n\,x^{\,n-1}$	(Power Rule)
$c\,f(x)$	$c\,f'(x)$	(Constant Multiple Rule)
$f(x)+g(x)$	$f'(x)+g'(x)$	(Sum Rule)
$f(x)-g(x)$	$f'(x)-g'(x)$	(Difference Rule)
$f(x)\,g(x)$	$f(x)\,g'(x)+f'(x)\,g(x)$	(Product Rule)
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}$	(Quotient Rule)
$f(g(x))$	$f'(g(x))\,g'(x)$	(Chain Rule)

Exponentials & Logarithms

Function	Derivative	
e^x	e^x	
a^x	$a^x \ln a$	
$\ln x$	$\dfrac{1}{x}$	(for $x>0$)
$\log_a x$	$\dfrac{1}{x\,\ln a}$	(for $x>0$)

Trigonometric Functions

Function	Derivative
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x\,\tan x$
$\csc x$	$-\csc x\,\cot x$

Inverse Trigonometric Functions

Function	Derivative	
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$	(for $\lvert x\rvert \le 1$)
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$	(for $\lvert x\rvert \le 1$)
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$	
$\cot^{-1} x$	$-\dfrac{1}{1+x^2}$	
$\sec^{-1} x$	$\dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}$	(for $\lvert x\rvert > 1$)
$\csc^{-1} x$	$-\dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}$	(for $\lvert x\rvert > 1$)

Chain Rule Forms

Function	Derivative	
$u(x)^n$	$n\,u(x)^{\,n-1}\,u'(x)$	
$e^{u(x)}$	$e^{u(x)}\,u'(x)$	
$a^{u(x)}$	$a^{u(x)}\ln(a)\,u'(x)$	
$\ln(u(x))$	$\dfrac{u'(x)}{u(x)}$	(for $u>0$)
$\log_a(u(x))$	$\dfrac{u'(x)}{u(x)\,\ln a}$	(for $u>0$)
$\sin(u(x))$	$\cos(u(x))\,u'(x)$	
$\tan(u(x))$	$\sec^2(u(x))\,u'(x)$	
$\sec(u(x))$	$\sec(u(x))\tan(u(x))\,u'(x)$	
$\cos^{-1}(u(x))$	$-\dfrac{u'(x)}{\sqrt{1-u(x)^2}}$	($\lvert u\rvert \le 1$)
$\cot^{-1}(u(x))$	$-\dfrac{u'(x)}{1+u(x)^2}$	

Implicit Differentiation

Function	Derivative, with $y=y(x)$
y	$\dfrac{dy}{dx}$
y^n	$n\,y^{\,n-1}\,\dfrac{dy}{dx}$
e^y	$e^y\,\dfrac{dy}{dx}$
a^y	$a^y\ln(a)\,\dfrac{dy}{dx}$
$\ln y$	$\dfrac{1}{y}\,\dfrac{dy}{dx}$
$\log_a y$	$\dfrac{1}{y\,\ln a}\,\dfrac{dy}{dx}$
$\sin y$	$\cos y\,\dfrac{dy}{dx}$
$\tan y$	$\sec^2 y\,\dfrac{dy}{dx}$
$\sec y$	$\sec y\,\tan y\,\dfrac{dy}{dx}$
$\cos^{-1} y$	$-\dfrac{1}{\sqrt{1-y^2}}\,\dfrac{dy}{dx}$
$\cot^{-1} y$	$-\dfrac{1}{1+y^2}\,\dfrac{dy}{dx}$

Workflow

(i) Differentiate both sides with respect to x.

(ii) Apply product/quotient/chain rules as needed.

(iii) Every time a y is differentiated, multiply by $\dfrac{dy}{dx}$. This comes from the chain rule, since we view y as a function of x.

(iv) Collect all $\dfrac{dy}{dx}$ terms on one side.

(v) Factor out $\dfrac{dy}{dx}$ and solve.