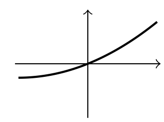
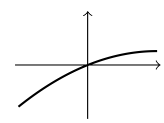
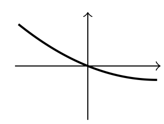
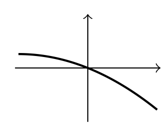
Relationships Among f, f', f''

1. What do f' and f'' say about f?

- $f'(x) > 0 \Rightarrow f$ increasing on that interval.
- $f'(x) < 0 \Rightarrow f$ decreasing on that interval.
- $|f'(x)|$ indicates steepness of f.
- $f''(x) > 0 \Rightarrow f$ concave up; f' is increasing.
- $f''(x) < 0 \Rightarrow f$ concave down; f' is decreasing.
- $|f''(x)|$ indicates how quickly slopes change.

2. Graph of $f(x)$ given f' and f''

$f'(x)$	$f''(x)$	**What f is doing**	**Schematic of $f(x)$**
> 0	> 0	Increasing; concave up (slopes becoming *more positive*).	
> 0	< 0	Increasing; concave down (slopes becoming *less positive*).	
< 0	> 0	Decreasing; concave up (slopes becoming *less negative*).	
< 0	< 0	Decreasing; concave down (slopes becoming *more negative*).	

3. Graph of $f'(x)$ given $f(x)$

Feature of f	**Implication for f'**
Horizontal tangent	$f'(x) = 0$.
Increasing region	$f'(x) > 0$.
Decreasing region	$f'(x) < 0$.
Concave up ($f'' > 0$)	f' is increasing.
Concave down ($f'' < 0$)	f' is decreasing.

Tips:

- Sketch slopes on f first (positive, negative, zero).
- Use concavity of f to decide whether f' rises or falls.

$f(x)$ $f'(x)$ $f''(x)$