6.1 Areas Between Curves

Theorem. Let f and g be continuous on $[a,b]$, and suppose $f(x) \geq g(x)$ for all x in $[a,b]$. The region between $y = f(x)$, $y = g(x)$, and the vertical lines $x = a$ and $x = b$ has area

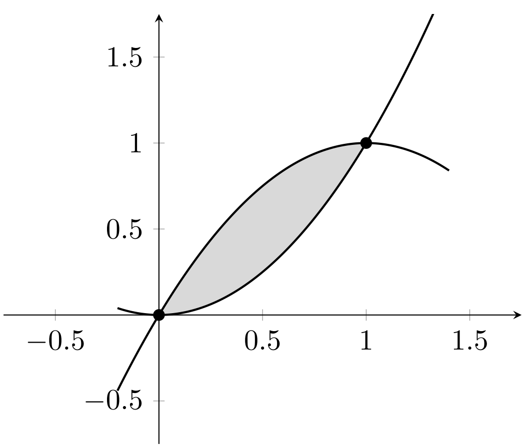
$$A = \int_a^b \big[f(x) - g(x)\big]\, dx.$$

Equivalently, if $y_T(x)$ is the top function and $y_B(x)$ is the bottom function on $[a,b]$, then

$$A = \int_a^b \big[y_T(x) - y_B(x)\big]\, dx.$$

Remark. Sometimes the vertical lines $x = a$ and $x = b$ are not given directly, and the region is just bounded by the curves $y = f(x)$ and $y = g(x)$. The endpoints a and b are the x-values where the curves intersect.

Example. Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.



Theorem. Let f and g be continuous on $[c, d]$, and suppose $f(y) \geq g(y)$ for all y in $[c, d]$. The region between $x = f(y)$ (right), $x = g(y)$ (left), and the horizontal lines $y = c$ and $y = d$ has area

$$A = \int_c^d \big[f(y) - g(y)\big]\,dy.$$

Equivalently, if $x_R(y)$ is the right function and $x_L(y)$ is the left function on $[c, d]$, then

$$A = \int_c^d \big[x_R(y) - x_L(y)\big]\,dy.$$

Remark. Sometimes the horizontal lines $y = c$ and $y = d$ are not given directly, and the region is just bounded by the curves $x = f(y)$ and $x = g(y)$. The endpoints c and d are the y-values where the curves intersect.

Example. Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.