5.5 The Substitution Rule

Theorem (The Substitution Rule)**.** If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du.$$

When we let $u = g(x)$, we have

$$du = g'(x)\, dx.$$

The symbols du and dx are called *differentials* and help us track how small changes in u relate to small changes in x; see Section 3.10 for more details.

Proof.

□

Example. Find $\int x^3 \cos(x^4 + 2)\ dx$.

Example. Evaluate $\int \sqrt{2x+1}\, dx$.

Example. Find $\displaystyle\int \frac{x}{\sqrt{1-4x^2}}\,dx$.

Example. Calculate $\displaystyle\int e^{5x}\,dx$.

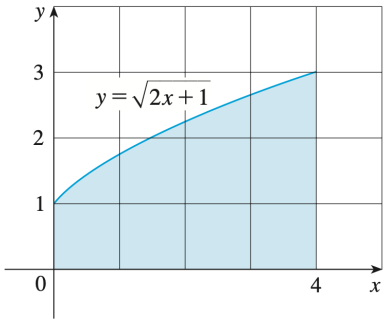
Example. Calculate $\int x^5\sqrt{1+x^2}\ dx$.

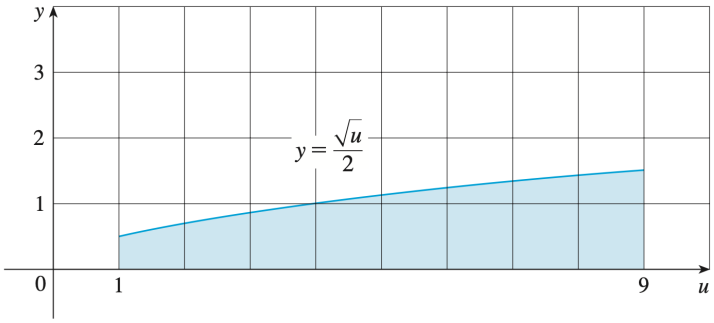
Example. Calculate $\int \tan x\ dx$.

Theorem (The Substitution Rule for Definite Integrals)**.** If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du.$$

Example. Evaluate $\displaystyle\int_0^4 \sqrt{2x+1}\, dx$.





Example. Evaluate $\displaystyle\int_1^2 \frac{dx}{(3-5x)^2}$.

Example. Calculate $\displaystyle\int_1^e \frac{\ln x}{x}\, dx$.