5.5 - Substitution Rule Worksheet – Solutions

1. Evaluate $\displaystyle\int \frac{x}{(x+5)^3}\,dx$. (Hint: use the substitution $u = x + 5$.)

Solution: Let $u = x + 5$, so that $x = u - 5$ and $dx = du$.

$$\begin{aligned}\int \frac{x}{(x+5)^3}\,dx &= \int \frac{u-5}{u^3}\,du \\ &= \int \left(u^{-2} - 5u^{-3}\right)du \\ &= -u^{-1} + \frac{5}{2}u^{-2} + C \\ &= \boxed{-\frac{1}{x+5} + \frac{5}{2(x+5)^2} + C}.\end{aligned}$$

2. Evaluate $\displaystyle\int \sin^4(\theta)\cos(\theta)\,d\theta$.

Solution: Let $u = \sin\theta$, so that $du = \cos\theta\,d\theta$.

$$\begin{aligned}\int \sin^4(\theta)\cos(\theta)\,d\theta &= \int u^4\,du \\ &= \frac{u^5}{5} + C \\ &= \boxed{\frac{\sin^5\theta}{5} + C}.\end{aligned}$$

3. Evaluate $\displaystyle\int x\sqrt{16 - x^2}\,dx$.

Solution: Let $u = 16 - x^2$, so that $du = -2x\,dx$ and $x\,dx = -\frac{1}{2}du$.

$$\begin{aligned}\int x\sqrt{16-x^2}\,dx &= \int \sqrt{u}\left(-\frac{1}{2}\,du\right) \\ &= -\frac{1}{2}\int u^{1/2}\,du \\ &= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C \\ &= \boxed{-\frac{1}{3}(16-x^2)^{3/2} + C}.\end{aligned}$$

4. Evaluate $\displaystyle\int 4t^3 e^{2-t^4}\,dt$.

Solution: Let $u = 2 - t^4$, so that $du = -4t^3\,dt$ and $4t^3\,dt = -du$.

$$\begin{aligned}\int 4t^3 e^{2-t^4}\,dt &= \int e^u(-du)\\ &= -\int e^u\,du\\ &= -e^u + C\\ &= \boxed{-e^{2-t^4} + C}.\end{aligned}$$

5. Evaluate $\displaystyle\int \frac{(\ln x)^4}{x}\,dx$.

Solution: Let $u = \ln x$, so that $du = \dfrac{1}{x}\,dx$.

$$\begin{aligned}\int \frac{(\ln x)^4}{x}\,dx &= \int u^4\,du\\ &= \frac{u^5}{5} + C\\ &= \boxed{\frac{(\ln x)^5}{5} + C}.\end{aligned}$$

6. Evaluate $\displaystyle\int_1^4 \frac{e^{1/x}}{x^2}\,dx$.

Solution: Let $u = \dfrac{1}{x}$, so that $du = -\dfrac{1}{x^2}\,dx$ and $\dfrac{1}{x^2}\,dx = -du$. When $x = 1$, we have $u = 1$, and when $x = 4$, we have $u = \frac{1}{4}$.

$$\begin{aligned}\int_1^4 \frac{e^{1/x}}{x^2}\,dx &= \int_1^4 e^{1/x}\,\frac{1}{x^2}\,dx\\ &= \int_1^{1/4} e^u(-du)\\ &= \int_{1/4}^1 e^u\,du\\ &= \left[e^u\right]_{1/4}^1\\ &= \boxed{e - e^{1/4}}.\end{aligned}$$

7. Evaluate $\displaystyle\int_0^{\ln 2} \frac{e^x}{1+e^{2x}}\,dx$. (Hint: use the substitution $u = e^x$.)

Solution: Let $u = e^x$, so that $du = e^x\,dx$ and $e^{2x} = u^2$. When $x = 0$, we have $u = 1$, and when $x = \ln 2$, we have $u = 2$.

$$\begin{aligned}\int_0^{\ln 2} \frac{e^x}{1+e^{2x}}\,dx &= \int_0^{\ln 2} \frac{e^x}{1+(e^x)^2}\,dx \\ &= \int_1^2 \frac{1}{1+u^2}\,du \\ &= \Big[\arctan u\Big]_1^2 \\ &= \arctan(2) - \arctan(1) \\ &= \boxed{\arctan(2) - \frac{\pi}{4}}.\end{aligned}$$

8. Suppose $\displaystyle\int_0^{36} g(u)\,du = 15$. Use an appropriate substitution to evaluate $\displaystyle\int_0^{6} x\,g(x^2)\,dx$.

Solution: Let $u = x^2$, so that $du = 2x\,dx$ and $x\,dx = \frac{1}{2}du$. When $x = 0$, we have $u = 0$, and when $x = 6$, we have $u = 36$.

$$\begin{aligned}\int_0^{6} x\,g(x^2)\,dx &= \int_0^{36} g(u)\,\frac{1}{2}\,du \\ &= \frac{1}{2}\int_0^{36} g(u)\,du \\ &= \frac{1}{2}\cdot 15 \\ &= \boxed{\frac{15}{2}}.\end{aligned}$$