

5.4 FTC Part 2, Indefinite Integrals

Theorem (Evaluation Theorem / FTC Part II). Let f be continuous on $[a, b]$ and let F be any antiderivative of f on $[a, b]$. Then

$$\int_a^b f(x) dx = F(b) - F(a).$$

Step 1. Partition the interval. Let n be a positive integer. Divide $[a, b]$ into n equal subintervals with endpoints

$$a = x_0 < x_1 < x_2 < \cdots < x_n = b,$$

each of length

$$\Delta x = \text{_____}.$$

Step 2. Write $F(b) - F(a)$ as a telescoping sum.

$$F(b) - F(a) =$$

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Step 3. Apply the Mean Value Theorem on each subinterval. On each subinterval $[x_{i-1}, x_i]$, the function F is continuous and differentiable, so by the Mean Value Theorem there exists a point x_i^* in $[x_{i-1}, x_i]$ with

$$F'(x_i^*) = \text{_____}.$$

In other words,

$$F(x_i) - F(x_{i-1}) =$$

Therefore,

$$F(b) - F(a) = \sum_{i=1}^n [F(x_i) - F(x_{i-1})] =$$

Step 4. Recognize a Riemann sum. Since f is continuous on $[a, b]$, the limit of these Riemann sums exists and equals the definite integral:

$$F(b) - F(a) =$$

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Notation: We often write this as

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a).$$

Example. Evaluate $\int_0^1 x^2 dx$.

Definition (Indefinite Integral). Let f be a function defined on an interval I . A function F is called an *antiderivative* of f on I if

$$F'(x) = f(x) \quad \text{for all } x \text{ in } I.$$

The *indefinite integral* of f with respect to x is the family of all antiderivatives of f and is written

$$\int f(x) dx = F(x) + C,$$

where $F'(x) = f(x)$ on I and C is an arbitrary constant.

Indefinite Integral Formulas

$$\int c f(x) dx = c \int f(x) dx$$

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

$$\int k dx = kx + C$$

$$\int \frac{1}{x} dx = \ln |x| + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int b^x dx = \frac{b^x}{\ln b} + C, \quad b > 0, b \neq 1$$

$$\int e^x dx = e^x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

$$\int \frac{1}{x^2+1} dx = \arctan x + C$$