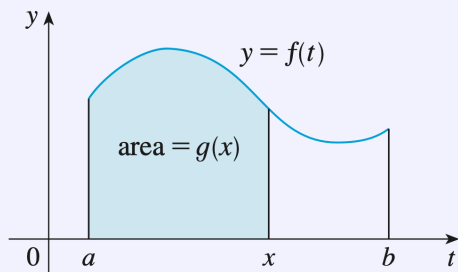
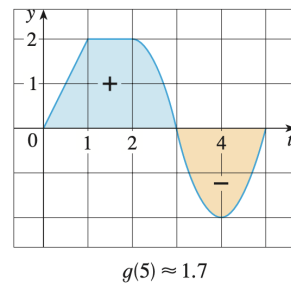
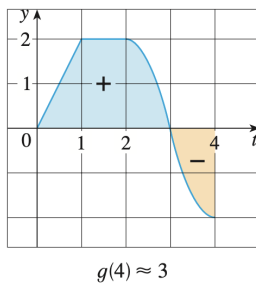
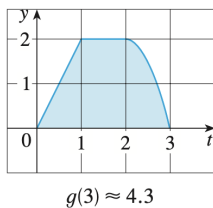
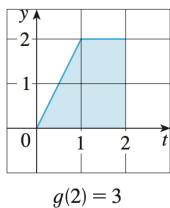
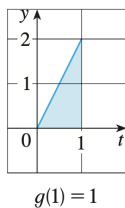
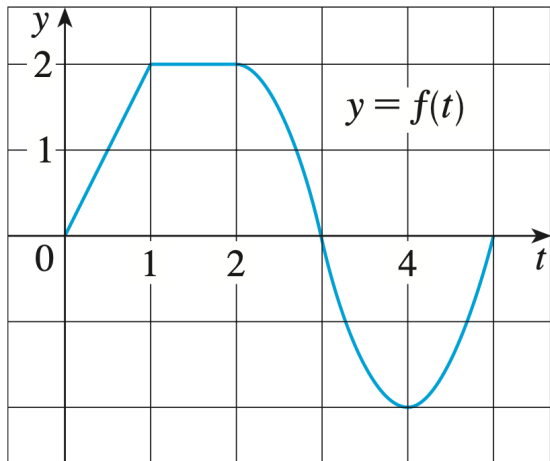


5.3 The Fundamental Theorem of Calculus

Definition. An **accumulation function** is a function of the form $g(x) = \int_a^x f(t)dt$. It represents the area under a curve $f(t)$ from a to x (i.e. the “area so far”).



Example. If f is the function whose graph is shown below and $g(x) = \int_0^x f(t)dt$, find the values of $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(4)$, and $g(5)$. Then sketch a rough graph of g .



Example. If $g(x) = \int_a^x f(t)dt$, where $a = 1$ and $f(t) = t^2$, find a formula for $g(x)$ and $g'(x)$.

Theorem (Fundamental Theorem of Calculus, Part 1). If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t) dt, \quad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b) , and $g'(x) = f(x)$.

Example. Find the derivative of the function $g(x) = \int_0^x \sqrt{1+t^2} dt$.

Example. Find $\frac{d}{dx} \int_1^{x^4} \sec t dt$.