5.2 The Definite Integral (Activity)

Step 1: Cut the interval. Split $[a, b]$ into n equal pieces:

$$\Delta x = \frac{b-a}{n}, \qquad x_i = a + i\,\Delta x \quad (i = 0, 1, \ldots, n).$$

Step 2: Pick one point in each small interval $[x_{i-1}, x_i]$. Common choices:

$$\text{Left: } x_{i-1} \qquad \text{Right: } x_i \qquad \text{Midpoint: } \frac{x_{i-1}+x_i}{2}.$$

Step 3: Add up rectangle areas.

$$L_n = \sum_{i=1}^{n} f(x_{i-1})\,\Delta x, \qquad R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x, \qquad M_n = \sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)\Delta x$$

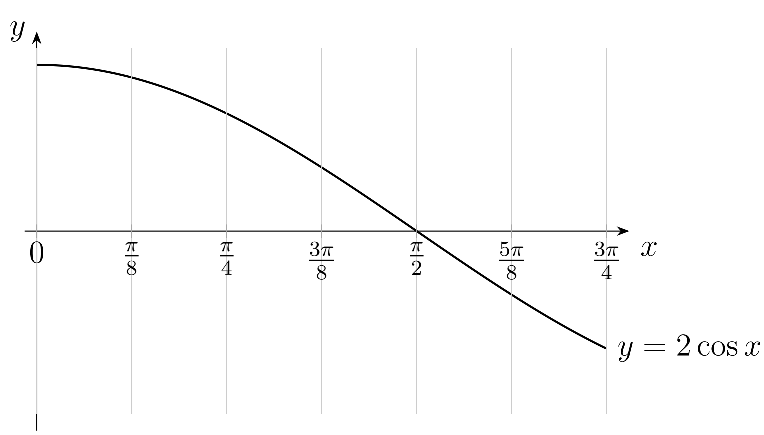
Definite Integral: The *net signed area* under f from a to b is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x, \quad \text{where } x_i^* \text{ in } [x_{i-1}, x_i] \text{ can be any sample point.}$$

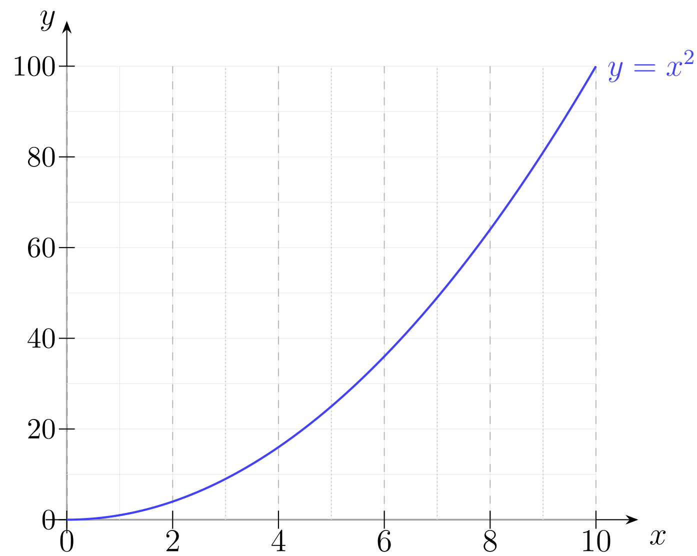
1. For $f(x) = 2\cos x$ on $[0, \frac{3\pi}{4}]$ with $n = 6$:

   (a) Compute L_6 to six decimal places.

   (b) Sketch the rectangles you used to compute L_6 (left endpoints).



2. Estimate $\int_2^{10} x^2\,dx$ by computing L_4 and R_4.

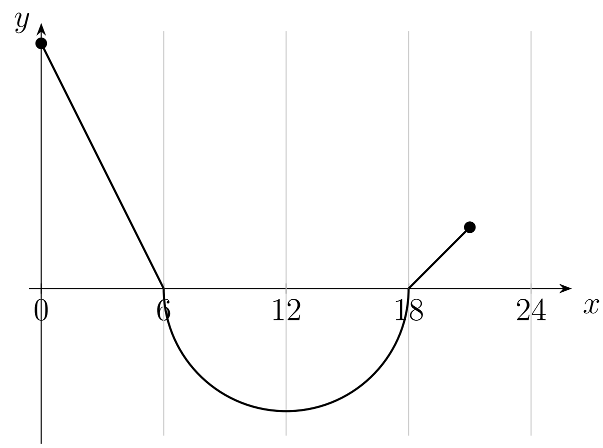


3. On $[2, 6]$, write $\lim_{n\to\infty} \sum_{i=1}^{n} \left(x_i^3 + x_i\right) \Delta x$ as a definite integral.

4. Write $\int_6^9 \left(x^2 + \frac{1}{x}\right) dx$ as a limit of Riemann sums using *right* endpoints.

5. Below, g is piecewise:

- $g(x) = 12 - 2x$ on $[0, 6]$,
- a semicircle of radius 6 *below* the x-axis on $[6, 18]$,
- linear from $(18, 0)$ to $(21, 3)$.



Evaluate:

(a) $\displaystyle\int_0^6 g(x)\,dx$

(b) $\displaystyle\int_6^{18} g(x)\,dx$

(c) $\displaystyle\int_0^{21} g(x)\,dx$

6. Suppose $\displaystyle\int_1^7 f(x)\,dx = 9.2$ and $\displaystyle\int_6^7 f(x)\,dx = 5.6$. Find $\displaystyle\int_1^6 f(x)\,dx$.

7. On $[1, 5]$ with $n = 8$, write M_8 for $f(x) = \sqrt{x}$. State Δx and the 8 midpoints.

8. Express $\displaystyle\int_2^{10} \big(3x^2 - 4x + 1\big)\, dx$ as a limit of right-endpoint sums. (Do not evaluate.)

9. Write $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \cdot \frac{1}{n}$ as a definite integral.