

4.9 Antiderivatives

Definition. A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I .

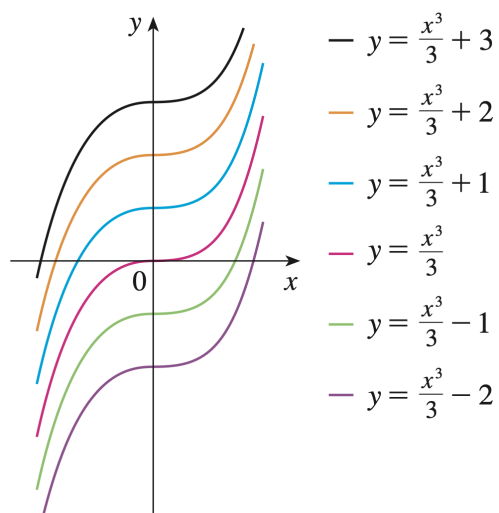
Theorem. If F is an antiderivative of f on an interval I , then the most general antiderivative of f on I is

$$F(x) + C,$$

where C is an arbitrary constant.

Example. What is an antiderivative of $f(x) = 1$?

Example. What is an antiderivative of $f(x) = x^2$?



Example. Find the most general antiderivative of $f(x) = x^n$ for any $n \neq -1$.

Example. Find the most general antiderivative of $f(x) = \sin(x)$.

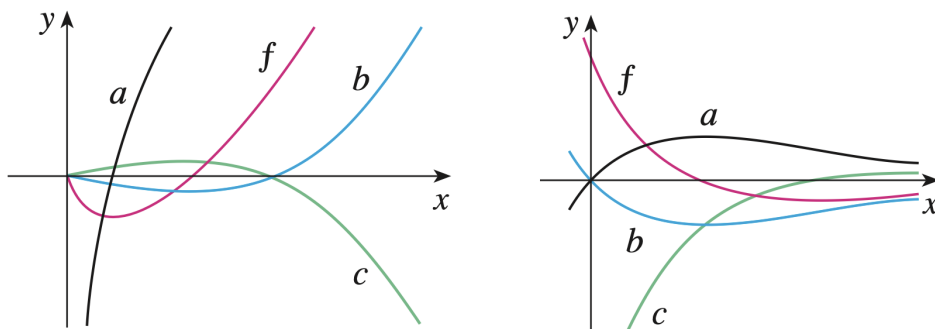
Example. Find all functions g such that $g'(x) = 4 \sin x + \frac{2x^5 - \sqrt{x}}{x}$.

Example. Find f if $f'(x) = e^x + 20(1 + x^2)^{-1}$ and $f(0) = -2$.

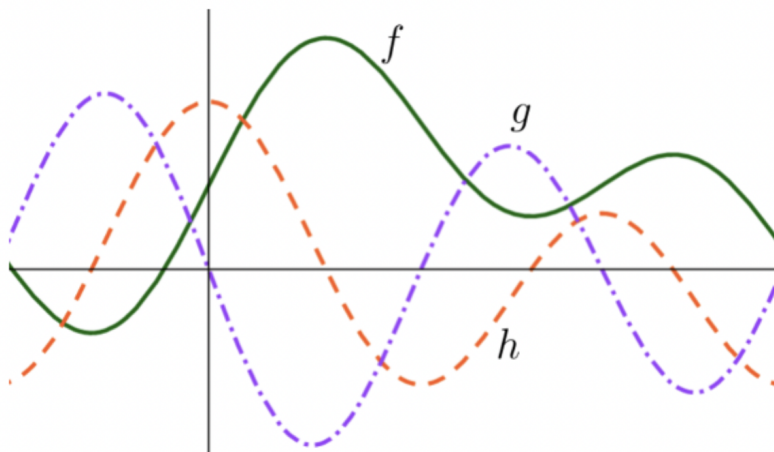
Example. Find f if $f''(x) = 12x^2 + 6x - 4$ if $f(0) = 4$ and $f(1) = 1$.

Example. A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $v(0) = -6$ cm/s and its initial displacement is $s(0) = 9$ cm. Find its position function $s(t)$.

Example. The graph of a function f is shown. Which graph is an antiderivative of f and why?



Example. The graph of a function, its derivative, and one of its antiderivatives is pictured below.



- (a) f is an antiderivative of g and h is the derivative of g .
- (b) h is an antiderivative of g and f is the derivative of g
- (c) g is an antiderivative of f and h is the derivative of f
- (d) h is an antiderivative of f and g is the derivative of f
- (e) f is an antiderivative of h and g is the derivative of h