4.7 Applied Optimization

Theorem (The Closed Interval Method.)**.** To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the critical numbers.
2. Find the values of f at the critical numbers of f in (a, b).
3. Find the values of f at the endpoints of the interval.
4. The largest of the values from Steps 1-3 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Theorem (The First Derivative Test)**.** Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' does not change sign at c, then f has no local maximum or minimum at c.

Theorem (Second Derivative Test)**.** Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

(c) If $f'(c) = 0$ and $f''(c) = 0$, then the test is inconclusive. In other words, at such a point there might be a maximum, there might be a minimum, or there might be neither.

Note: If either $f'(c)$ or $f''(c)$ does not exist, the test cannot be applied.

Theorem (First Derivative Test for Absolute Extreme Values)**.** Suppose that c is a critical number of a continuous function f.

(a) If $f'(x) > 0$ when $x < c$ and $f'(x) < 0$ when $x > c$, then $f(c)$ is the absolute maximum of f.

(b) If $f'(x) < 0$ when $x < c$ and $f'(x) > 0$ when $x > c$, then $f(c)$ is the absolute minimum of f.

Example. Find two numbers whose difference is 100 and whose product is a minimum.

Example (Maximizing Revenue)**.** A store has been selling 200 computers per week at \$350 each. A market survey indicates that for each decrease in price of \$10, the number of units sold will increase by 20 per week. What price will maximize the store's weekly revenue?