4.7 Applied Optimization

Theorem (The Closed Interval Method.)**.** To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the critical numbers.
2. Find the values of f at the critical numbers of f in (a, b).
3. Find the values of f at the endpoints of the interval.
4. The largest of the values from Steps 1-3 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Theorem (The First Derivative Test)**.** Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' does not change sign at c, then f has no local maximum or minimum at c.

Theorem (Second Derivative Test)**.** Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

(c) If $f'(c) = 0$ and $f''(c) = 0$, then the test is inconclusive. In other words, at such a point there might be a maximum, there might be a minimum, or there might be neither.

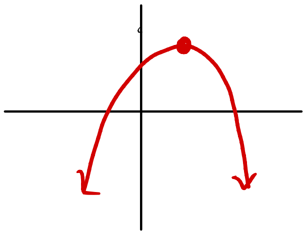
Note: If either $f'(c)$ or $f''(c)$ does not exist, the test cannot be applied.

New!!

Theorem (First Derivative Test for Absolute Extreme Values)**.** Suppose that c is a critical number of a continuous function f.

(a) If $f'(x) > 0$ when $x < c$ and $f'(x) < 0$ when $x > c$, then $f(c)$ is the absolute maximum of f.

(b) If $f'(x) < 0$ when $x < c$ and $f'(x) > 0$ when $x > c$, then $f(c)$ is the absolute minimum of f.



f' changes sign once
on an open interval
⇓
ABSOLUTE max/min!

x and y

Example. Find two numbers whose difference is 100 and whose product is a minimum.

① Given: $x - y = 100$

② Want: Minimize the product $x \cdot y$

Need a function of one variable. Since $y = x - 100$ want to minimize

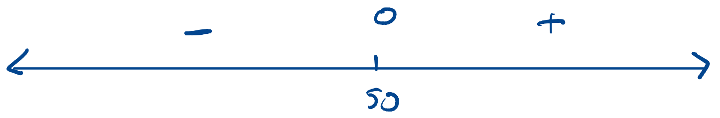
$$f(x) = x(x-100) = x^2 - 100x$$

Critical Numbers: $f'(x) = 2x - 100$

$$0 = 2x - 100$$

$\Rightarrow$ one critical number when $x = 50$

Check max/min/neither:



· The first derivative test says this is a LOCAL min

· The F.D.T.A.E.V. says this is also the ABSOLUTE min

$\Rightarrow$ $x = 50$ and $y = -50$, minimum product is -2500

Example (Maximizing Revenue)**.** A store has been selling 200 computers per week at \$350 each. A market survey indicates that for each decrease in price of \$10, the number of units sold will increase by 20 per week. What price will maximize the store's weekly revenue?

Strategy:

x is units sold, $p(x)$ is price the store can charge

- Find the demand function $p(x)$
- Find the revenue function $R(x)$ and maximize it

Demand Function:

x	$p(x)$
200	\$350
220	\$340
240	\$330
260	\$320

$$\boxed{p(x) = 450 - \tfrac{1}{2}x}$$

Use point-slope form

$$y - y_1 = m(x - x_1)$$

$$m = \frac{\Delta y}{\Delta x} = \frac{340 - 350}{220 - 200} = \frac{-10}{20} = -\frac{1}{2}$$

$$y - 350 = -\tfrac{1}{2}(x - 200)$$

$$y = -\tfrac{1}{2}x + 100 + 350$$

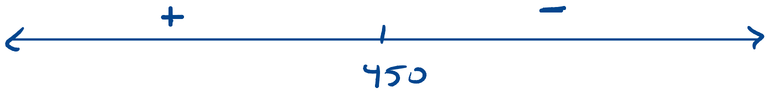
Revenue Function:

Revenue = Units Sold × Price

$$R(x) = x \cdot p(x) = x\left(450 - \tfrac{1}{2}x\right) = 450x - \tfrac{1}{2}x^2$$

Critical Numbers:

$R'(x) = 450 - x$, so one critical number at $x = 450$



By F.D.T.A.E.V., this is an absolute maximum

Answer: selling 450 units at a price of $p(450) = \$225$ each will maximize revenue