L'Hôpital's Rule — Type $0/0$ and ∞/∞

A. For each limit, identify the *indeterminate form* by direct substitution. Write $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\frac{-\infty}{\infty}$, $\frac{\infty}{-\infty}$, $\frac{-\infty}{-\infty}$, or determinate.

1. $\lim_{x\to 0} \dfrac{\sin x}{x}$

2. $\lim_{x\to 0} \dfrac{1-\cos x}{x}$

3. $\lim_{x\to \infty} \dfrac{\ln x}{\sqrt{x}}$

4. $\lim_{x\to \infty} \dfrac{e^{-x}}{x^3}$

5. $\lim_{x\to 0^+} \dfrac{\ln x}{1/x}$

6. $\lim_{x\to 0^+} \dfrac{\csc x}{-\cot x}$

B. Evaluate the following limits.

1. $\lim_{x\to 0} \dfrac{\sin(3x)}{x}$

2. $\lim_{x\to 1} \dfrac{x-\sqrt{x}}{x-1}$

3. $\lim_{x\to \pi^-} \dfrac{\sin x}{1-\cos x}$.

4. $\lim_{x\to 0} \dfrac{\tan x - x}{x^3}$

5. $\lim_{x\to 0} \dfrac{\ln(1+2x) - 2x}{x^2}$

6. $\lim_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$

7. $\lim_{x\to \infty} \dfrac{e^x}{x^2}$

8. $\lim_{x\to 0^+} \dfrac{\ln x}{\tan(x - \frac{\pi}{2})}$

C. In the graph below, the blue curve is $f(x)$ and the red curve is $g(x)$. Both pass through $(2, 0)$ and are differentiable at $x = 2$. The dashed lines are the tangent lines at $x = 2$. Evaluate $\lim_{x\to 2} \dfrac{f(x)}{g(x)}$.