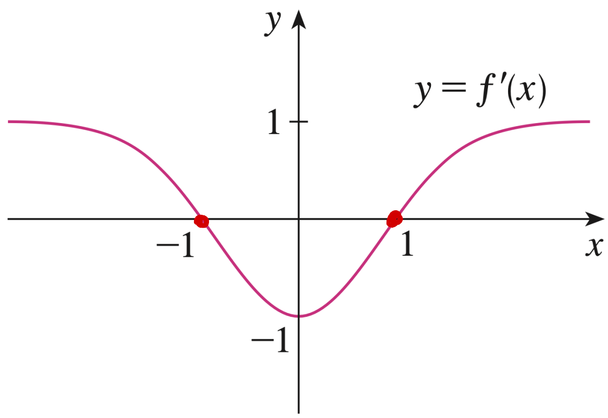
4.3 What Does f' Say About f?

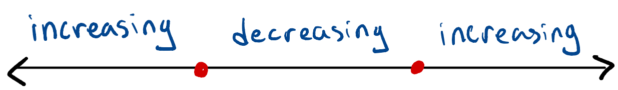
Theorem.

- If $f'(x) < 0$ on an interval, then f is decreasing on that interval.
- If $f'(x) > 0$ on an interval, then f is increasing on that interval.

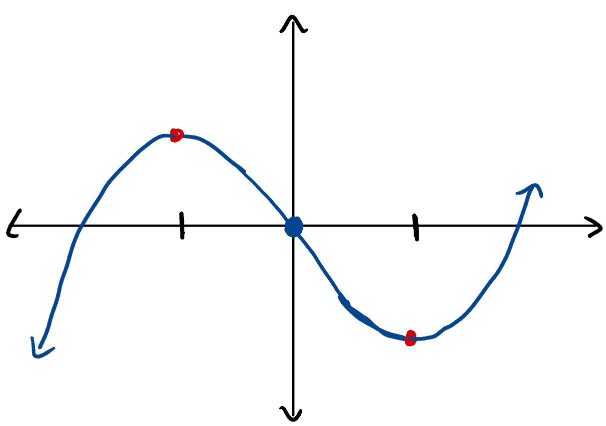
Example. The graph of the derivative f' of a function f is shown below. If $f(0) = 0$, sketch a possible graph of f.



$f(x)$



$f(x)$



$f(0) = 0$

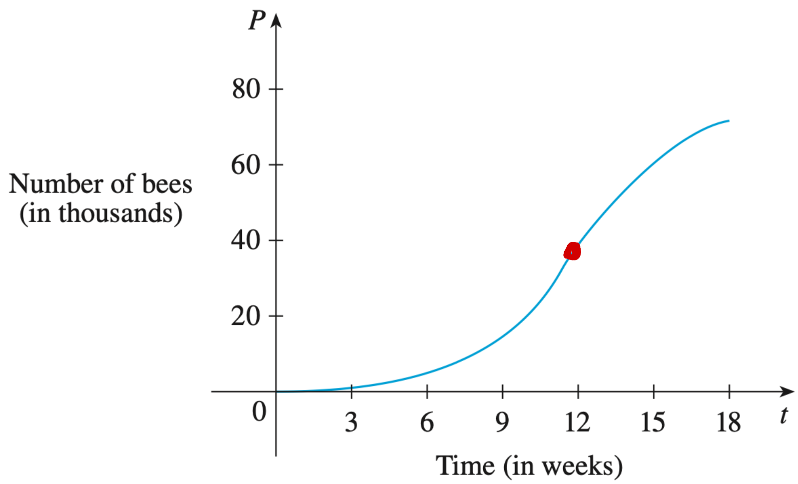
$f'(0) = -1$

Theorem.

- If $f''(x) < 0$ on an interval, then f is concave down on that interval.
- If $f''(x) > 0$ on an interval, then f is concave up on that interval.

Example. Below is a population graph for honeybees raised in an apiary.

- Over what intervals is P concave upward or concave downward?
- When is the rate of population increasing the fastest?



① Concave up $(0, 12)$

Concave down $(12, \infty)$

② Fastest rate of increase is at the most steep tangent line (pos. slope)

(Why is this where f changes concavity???)

Example. Sketch a possible graph of a function f that satisfies the following conditions:

- $f'(x) > 0$ on $(-\infty, 1)$, $f'(x) < 0$ on $(1, \infty)$.
- $f''(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$, $f''(x) < 0$ on $(-2, 2)$.
- $\lim_{x \to -\infty} f(x) = -2$, $\lim_{x \to \infty} f(x) = 0$.