## 4.1 Maximum and Minimum Values

**Definition.** Let $c$ be a number in the domain $D$ of a function $f$. Then $f(c)$ is the

- Absolute maximum value of $f$ on $D$ if $f(c) \geq f(x)$ for all $x$ in $D$.
- Absolute minimum value of $f$ on $D$ if $f(c) \leq f(x)$ for all $x$ in $D$.

**Definition.** Let $c$ be a number in the domain $D$ of a function $f$. Then $f(c)$ is a

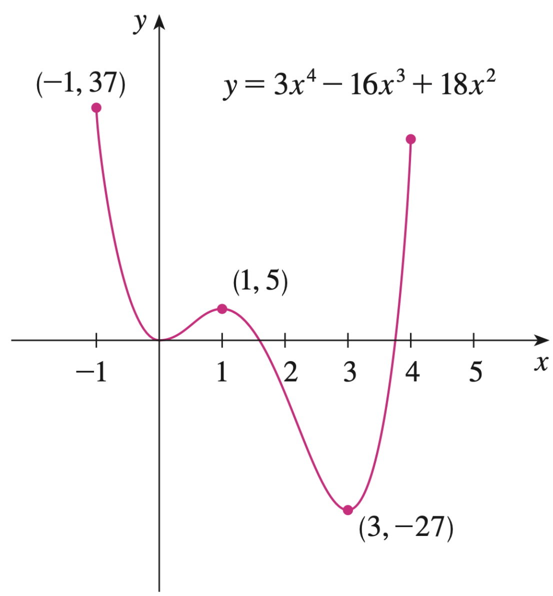
- Local maximum value of $f$ if $f(c) \geq f(x)$ when $x$ is near $c$.
- Local minimum value of $f$ if $f(c) \leq f(x)$ when $x$ is near $c$.

"Near $c$" means on some open interval $I$ containing $c$. In particular, a local maximum or minimum cannot occur at an endpoint of an interval.

**Example.** The graph of the function

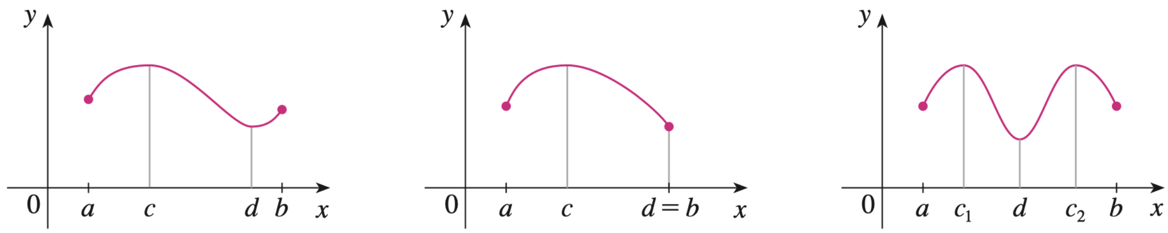
$$f(x) = 3x^4 - 16x^3 + 18x^2 \qquad -1 \leq x \leq 4$$

is graphed below. Find any maxima or minima.



**Theorem** (Extreme Value Theorem)**.** If $f(x)$ is continuous on a closed interval $[a, b]$, then $f(x)$ attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers $c$ and $d$ in $[a, b]$.

**Example.**

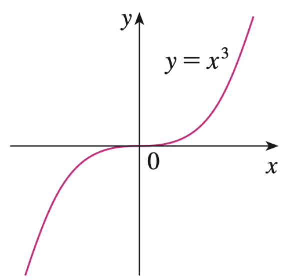


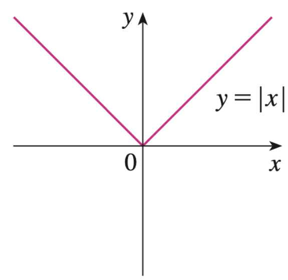

**Example.** Are absolute extrema guaranteed if the function *not continuous* on the closed interval?

**Example.** Are absolute extrema guaranteed for a continuous function on an *open interval*?

**Theorem.** If $f$ has a local maximum or minimum at $c$, and if $f'(c)$ exists, then $f'(c) = 0$.

**Example.** Why do we have to be careful with the above theorem?



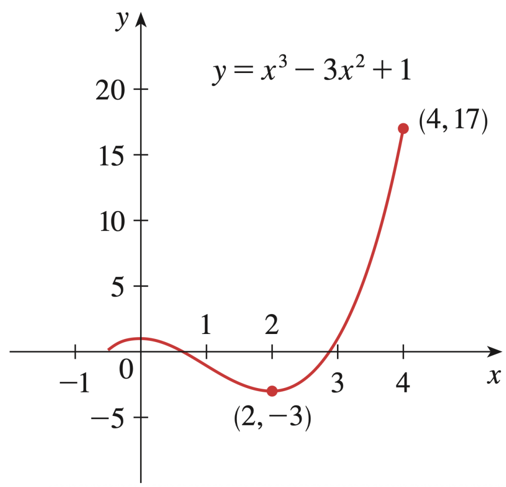

**Definition.** A critical number of a function $f$ is a number $c$ in the domain of $f$ such that either $f'(c) = 0$ or $f'(c)$ does not exist.

**Example.** Find the critical numbers of $f(x) = x^{3/5}(4 - x)$.

**The Closed Interval Method.** To find the *absolute* maximum and minimum values of a continuous function $f$ on a closed interval $[a, b]$:

1. Find the critical numbers.
2. Find the values of $f$ at the critical numbers of $f$ in $(a, b)$.
3. Find the values of $f$ at the endpoints of the interval.
4. The largest of the values from Steps 1-3 is the absolute maximum value; the smallest of these values is the absolute minimum value.

**Example.** Find the absolute minimum and maximum values of the function $f(x) = x^3 - 3x^2 + 1$ on the interval $[-\frac{1}{2}, 4]$.



**Example.** Find the absolute minimum and maximum values of the function $f(x) = x - 2\sin x$ on the interval $[0, 2\pi]$.