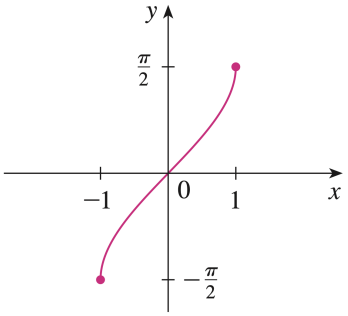
3.6 Inverse Trigonometric Functions and Their Derivatives

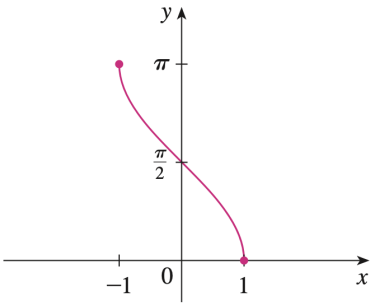
$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\left(\csc^{-1} x\right) = -\frac{1}{|x|\sqrt{x^2-1}},$$

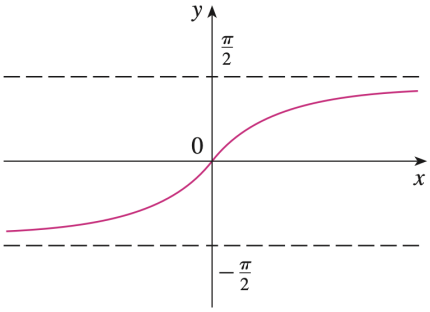
$$\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}},$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}, \qquad \frac{d}{dx}\left(\cot^{-1} x\right) = -\frac{1}{1+x^2}.$$

Question. What are the functions $\sin^{-1}(x)$, $\cos^{-1}(x)$, and $\tan^{-1}(x)$? What are the domains and ranges of these functions?







Example. Evaluate

(a) $\sin^{-1}(\frac{1}{2})$

(b) $\tan(\arcsin(\frac{1}{3}))$

Theorem. Show that $\dfrac{d}{dx}\sin^{-1}(x) = \dfrac{1}{\sqrt{1-x^2}}$.

Proof.

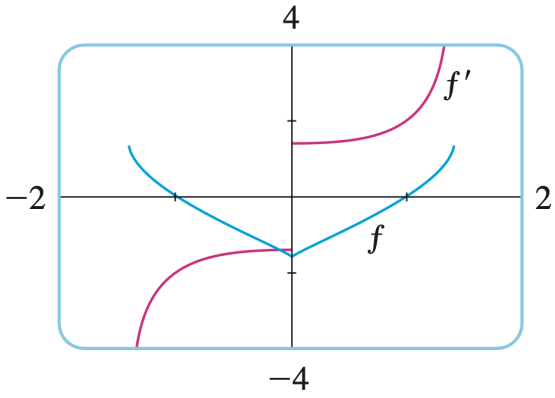
□

Example. If $f(x) = \sin^{-1}(x^2 - 1)$, find

(a) The domain of f

(b) The derivative f'

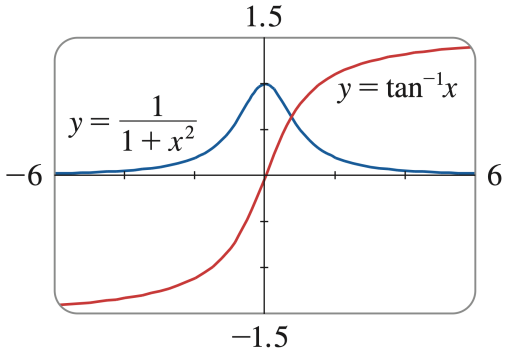
(c) The domain of f'



Example. Differentiate $y = \cos^{-1}(e^{2x})$.

Theorem. Show that $\dfrac{d}{dx}\tan^{-1}(x) = \dfrac{1}{1+x^2}$.

Proof.



□

Example. Differentiate $y = \dfrac{1}{\tan^{-1} x}$.

Example. Differentiate $y = x \arctan \sqrt{x}$.