

3.6 Logarithmic Functions and Their Derivatives

$$\frac{d}{dx}(\log_b(x)) = \frac{1}{x \ln(b)}$$

Proof.

□

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

Proof.

□

Example. Differentiate $y = \ln(x^3 + 1)$.

In general, if we have a composition of functions, the chain rule looks like:

Leibniz Notation

$$\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx}$$

Prime Notation

$$\frac{d}{dx}[\ln g(x)] = \frac{g'(x)}{g(x)}$$

Example. Find $\frac{d}{dx} \ln(\sin x)$.

Example. Differentiate $f(x) = \sqrt{\ln x}$

Example. Differentiate $f(x) = \log_{10}(2 + \sin x)$.

Example. Find $\frac{d}{dx} \ln \frac{x+1}{\sqrt{x-2}}$.

Steps in Logarithmic Differentiation

1. Take natural logarithms of both sides of an equation $y = f(x)$ and use the Laws of Logarithms to expand the expression.
2. Differentiate implicitly with respect to x .
3. Solve the resulting equation for y' and replace y by $f(x)$.

Example. Differentiate $y = \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$

Example. Differentiate $y = x^{\sqrt{x}}$.