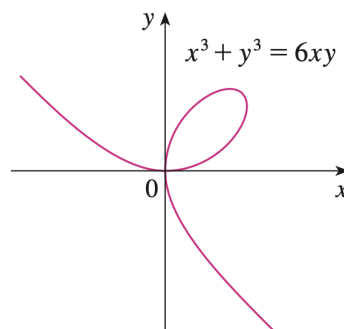
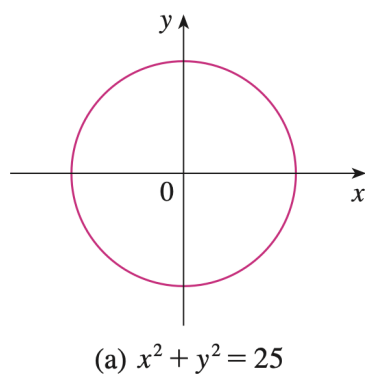


3.5 Implicit Differentiation

Question. What is the difference between a function and a relation?



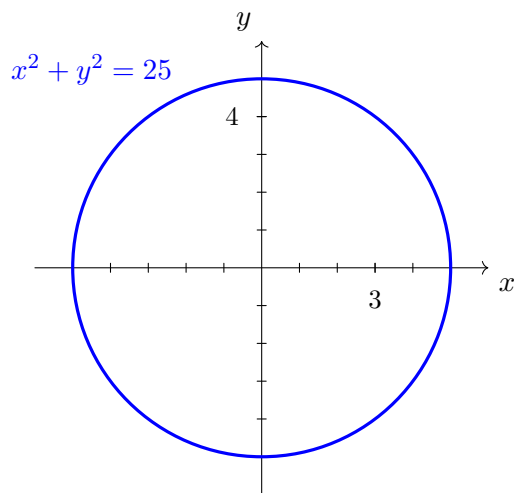
Implicit Differentiation: When y is not explicitly written as a function of x , we can still find the slope $\frac{dy}{dx}$ by differentiating *both sides with respect to x* and using the Chain Rule. We view y as an *unknown function $y(x)$* .

1. **Start from the relation.** Write the given equation relating the variables x and y .
2. **Differentiate both sides with respect to x .** Apply derivative rules as needed. Every time you differentiate a term involving y , multiply by a factor $y' = \frac{dy}{dx}$ via the Chain Rule.
3. **Collect y' -terms.** Move all terms containing y' to one side and all other terms to the opposite side.
4. **Factor and solve for y' .** Factor out y' and isolate it to obtain an explicit formula $y' = \frac{dy}{dx}$ in terms of x and y .

Example.

(a) If $x^2 + y^2 = 25$, find $\frac{dy}{dx}$.

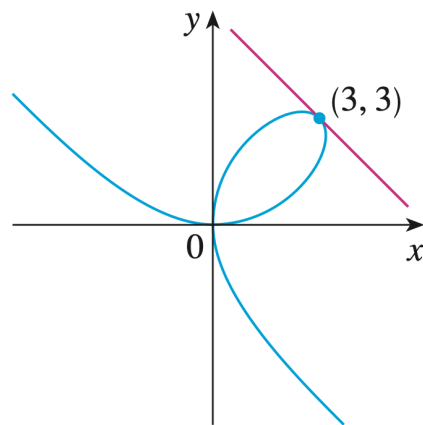
(b) Find an equation of the tangent line to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.



Example.

(a) Find y' if $x^3 + y^3 = 6xy$

(b) Find an equation of the tangent line to the folium of Descartes $x^3 + y^3 = 6xy$ at the point $(3, 3)$



Example. Find y' if $\sin(x + y) = y^2 \cos(x)$.

Example. Find y' if $x^4 + y^4 = 16$.

