3.4 The Chain Rule

Theorem. If $u = g(x)$ is differentiable at x and $y = f(u)$ is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x, and F' is given by the product:

Leibniz Notation	**Prime Notation**
$\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$	$F'(x) = f'\big(g(x)\big)\, g'(x)$

Example. Find $F'(x)$ if $F(x) = \sqrt{x^2 + 1}$.

Example. Differentiate $y = \sin(x^2)$.

Example. Differentiate $y = \sin^2(x)$.

Example. Differentiate $y = (x^3 - 1)^{100}$

Example. Find $f'(x)$ if $f(x) = \dfrac{1}{\sqrt[3]{x^2+x+1}}$.

Example. Find the derivative of the function $g(t) = \left(\dfrac{t-2}{2t+1}\right)^9$

Example. Differentiate $y = (2x + 1)^5(x^3 - x + 1)^4$

Example. Differentiate $y = e^{\sin x}$

Example. Differentiate $f(x) = \sin(\cos(\tan x))$.

Example. Differentiate $y = e^{\sec 3\theta}$.

Theorem. Let $a > 0$ with $a \neq 1$. Then

$$\frac{d}{dx}(a^x) \;=\; a^x \ln a.$$

Proof.

□

Example. Differentiate $g(x) = 2^x$

Example. Differentiate $g(x) = 5^{x^2}$